Sample Problems for the First Midterm

- The exam is on Monday, 10/27, 1:10 pm – 2:00 pm.
- In each problem, you have to show every step of your calculation.

Basic notions:

1. Find the limits:
   (1) \( \lim_{x \to \frac{\pi}{4}} 3 \cos 2x \).
   (2) \( \lim_{x \to 0} x^2 + 1000 \).
   (3) \( \lim_{x \to 1} \frac{x^3}{x+5} \).
   (4) \( \lim_{x \to 1} \frac{x^2+x-2}{x^3-1} \).
   (5) \( \lim_{x \to 1^+} \frac{\sqrt{x+8} - 3}{\sqrt{x+8} - 3} \).
   (6) \( \lim_{x \to 0} \frac{3 \cos 2x - \sin x - 3}{x} \).

2. Are following functions continuous?
   (1) \( f(x) = x^2 + \sin 2x \).
   (2) \( y = 1 \) if \( x \) is a rational number and \( y = \cos x \) if \( x \) is irrational.

3. Find the derivatives:
   (1) \( \sin x + \tan x \).
   (2) \( 3x(x^3 + 2003)^9 \).
   (3) \( \sec(x^{-\frac{2}{3}} + 1) \).

Application problems:

4. Find the derivative of \( y \) if \( x^2 + 3y^2x - y^3 = 3 \). Find the tangent line at \((1, 1)\).

5. Find the velocity and the acceleration of a object with a position function \( s = 3t^4 - 2t + 1 \) at \( t = 1 \).