Answers to Selected Sample Problems for the First Exam

2. (1) \( y = x + \frac{c}{x} \)  (2) \( y = \frac{x + c}{\sin x} \).

3. (1) \( y = \frac{1}{2} + \frac{1}{2}e^{-x^2} \)  (2) \( y = \frac{x - \frac{7}{x}}{\sin x} \).

4.

(1) Equation: \( \frac{dS}{dt} = 0.06S - 0.1, \ S(0) = 1 \).

(2) Solution: \( S(t) = e^{0.06t} - \frac{10}{6}(e^{0.06t} - 1) \).

(3) Let \( e^{0.06t} - \frac{10}{6}(e^{0.06t} - 1) = 0 \) and solve for \( t \), get

\[
    t = \frac{\ln 2.5}{0.06} = 15.27.
\]

So the money will last for about 15.27 years.

5.

(1) Equation: \( \frac{dT}{dt} = -k(T - 70), \ T(0) = 35, \ T(10) = 40 \).

(2) Solution: \( T = 35e^{-kt} + 70(1 - e^{-kt}) \).

(3) Find \( k \) using \( T(10) = 40 \): Let \( 40 = 35e^{-10k} + 70(1 - e^{-10k}) \) and get

\[
    k = \frac{1}{10} \ln \frac{35}{30}.
\]

(4) Let \( 35e^{-kt} + 70(1 - e^{-kt}) = 45 \), get

\[
    t = \frac{1}{k} \ln \frac{35}{25} = 10 \frac{\ln(35/25)}{\ln(35/30)} = 21.83.
\]

So it takes about 11.83 minutes to warm up the beer from 40°F to 45°F.