Sample Problems for the First Exam

- The exam is on Thursday, 02/15, 9:40 pm - 11:00 pm.
- In each problem, you have to show every step of your calculation.

Basic notions:
1. Verify that \( \alpha(t) = (t, t^2, \sqrt{t}) \) is a regular curve for \( t > 0 \). Is it regular at \( t = 0 \)?

2. Is \( X(r, \theta) = (r \cos \theta, r \sin \theta, r) \) a regular surface for \( r > 0 \)? Is it regular at \( r = 0 \)?

Geometric Calculations:
3. Let \( \alpha(t) = (t, \sin t, \cos t) \)
   (1) Find the tangent vector and the normal vector.
   (2) Find the binormal vector.
   (3) Find the curvature and torsion.
   (4) Find the length of \( \alpha([0, 2\pi]) \).

4. Find the first fundamental form of \( X(u, v) = (u, v, uv) \). Find the area of \( S \) with \( u^2 + v^2 \leq 1 \).