Answers to Selected Sample Problems for the First Exam

2. (1) $y = x + \dfrac{c}{x}$. (2) $y = \dfrac{x + c}{\sin x}$.

3. (1) $y = \dfrac{1}{2} + \dfrac{1}{2}e^{-x^2}$. (2) $y = \dfrac{x - \frac{\pi}{2}}{\sin x}$.

4.

(1) Equation: $\dfrac{dS}{dt} = 0.06S - 0.1,\ S(0) = 1.$

(2) Solution: $S(t) = e^{0.06t} - \dfrac{10}{6}(e^{0.06t} - 1).$

(3) Let $e^{0.06t} - \dfrac{10}{6}(e^{0.06t} - 1) = 0$ and solve for t, get

$$t = \frac{\ln 2.5}{0.06} = 15.27.$$

So the money will last for about 15.27 years.

5.

(1) Equation: $\dfrac{dT}{dt} = -k(T - 70),\ T(0) = 35,\ T(10) = 40.$

(2) Solution: $T = 35e^{-kt} + 70(1 - e^{-kt})$.

(3) Find k using $T(10) = 40$: Let $40 = 35e^{-10k} + 70(1 - e^{-10k})$ and get

$$k = \frac{1}{10}\ln\frac{35}{30}.$$

(4) Let $35e^{-kt} + 70(1 - e^{-kt}) = 45$, get

$$t = \frac{1}{k}\ln\frac{35}{25} = 10\frac{\ln(35/25)}{\ln(35/30)} = 21.83.$$

So it takes about 11.83 minutes to warm up the beer from 40°F to 45°F.