Math 10B Final Exam Winter 2008

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Math 10B Final Exam

• This is a close book exam. The total points are 100+5.
• In each problem, you have to show every step of your calculation.

Name: _______________________
ID Number: ___________________

1. (12 points)

(a) Find the double integral \( \iint_D f \, dA \) for \( f(x, y) = x^2y \) with \(-1 \leq x \leq y \leq 1\).

(b) Find the triple integral \( \iiint_W f \, dV \) for \( f(x, y, z) = x^2y + z \) with \( x^2 \leq y \leq z \leq 1 \).
2. (10 points) Find \( \iiint_W (\tan(2x-y) + y + z^3) \, dV \) with \( 2x-y, y, z \in [0, 1] \).
3. (8 points) Find the differential of $x^8 dy + z^{10} dz$. 
4. (10 points) Find the volume of the region $x^2 + y^2 \leq z \leq 8 - (x^2 + y^2)$. 
5. (10 points) Find the line integral

\[ \int_{C} yz \, dx + xz \, dy + xy \, dz \]

where \( C(t) = (\cos t, \sin t, t), 0 \leq t \leq \pi \).
6. (10 points) Let \( \Phi(u, v) = (u^2v, u + v, \sin v), 0 \leq u, v \leq 1 \). Is \( \Phi \) regular?
Find the tangent plane at \((u, v) = (1, 0)\).
7. (10 points) Find the surface integral

\[ \int \int_S \mathbf{F} \cdot d\mathbf{S} \]

where \( \mathbf{F} = (x, y, z^2) \), \( S : z^2 \leq x^2 + y^2, x^2 + y^2 + z^2 = 2 \).
8. (10 points) Find the line integral \( \int_C (3x^{10} + y)dx + (y^{2008} - 10x)dy + z^{10}dz \) where \( C \) is \( y^2 = z, 4 = x^2 + y^2 \) and counterclockwise with respect to the xy plane.
9. (10+5 points) Find the surface integral

\[ \iint_S (x + 10y)dydz + (y + 2008xz)dzdx + (z + y)dxdy \]

where \( S : x^2 + y^2 + z^2 = 1 \). (Bonus problem) Find

\[ \iint_S (f(x)dxdy + g(y)dydz). \]
10. (10 points) Is $\mathbf{F} = (3x^2y + zy, x^3 + xz, xy)$ conservative? If so, explain, then find the antiderivative of $\mathbf{F}$. Apply it to the line integral

$$\int_{C} \mathbf{F} \cdot ds$$

with $C$ the curve $(t, t^2, t^{10^{20081}}) \ t \in [0, 1]$. 