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Math 572 – Final Exam
December 14, 2006

1. In **Euclidean** geometry, prove that the sum of the angles of a triangle is 180° (you may use results up to IV.9 in Davis).

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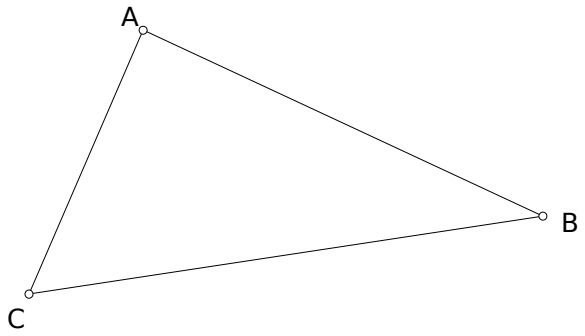
2. In **Neutral** geometry, prove that if $AB > AC$, then $\angle ABC < \angle ACB$ (you may use results up to III.5 in Davis).



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3. Consider the four-point geometry described by the following axioms:
   Undefined terms: point, line, on.
   Axiom 1: There exist exactly four points.
   Axiom 2: Two distinct points are on exactly one line.
   Axiom 3: Each line lies on exactly two points.

   a. Construct a model for this geometry

   b. Obtain an axiomatic system for the four-line geometry by dualizing the axioms for four-point geometry. *Note*: you don't have to prove the dual statements, nor do you have to draw a model. You just need to state the axioms.

Name: _____

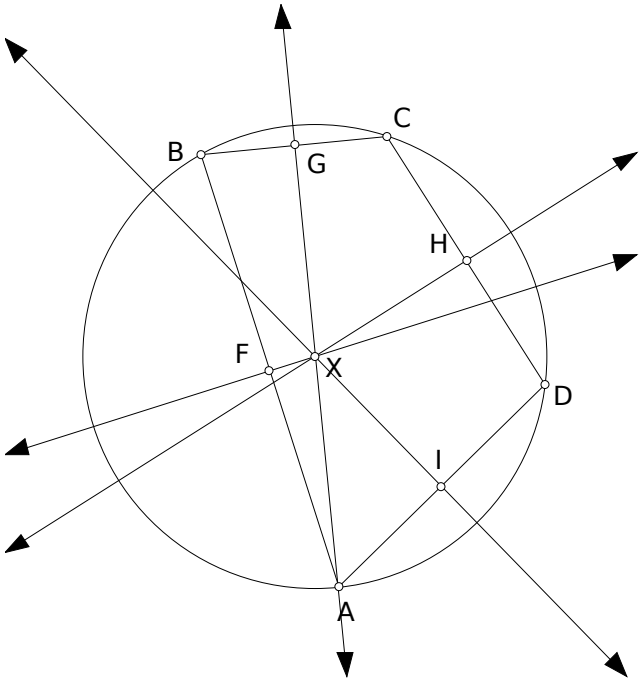
4. In **Euclidean** geometry, prove that the diagonals of a rhombus are perpendicular (you may use any results from Davis).

Name: _____

5. Is the following a valid proof in **Euclidean** geometry? If the proof is incorrect, indicate the errors. (Remember, you can give an incorrect proof for a correct theorem).

Theorem: The perpendicular bisectors of a quadrilateral are concurrent.

Proof: Given a quadrilateral, $\square ABCD$, draw a circle with center X circumscribing the quadrilateral. Next, construct the midpoints F, G, H, and I of sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ respectively, and then construct the perpendicular bisectors for each side. Now since the sides of the quadrilateral are chords of the circle, the perpendicular bisectors all pass through the center of the circle, X



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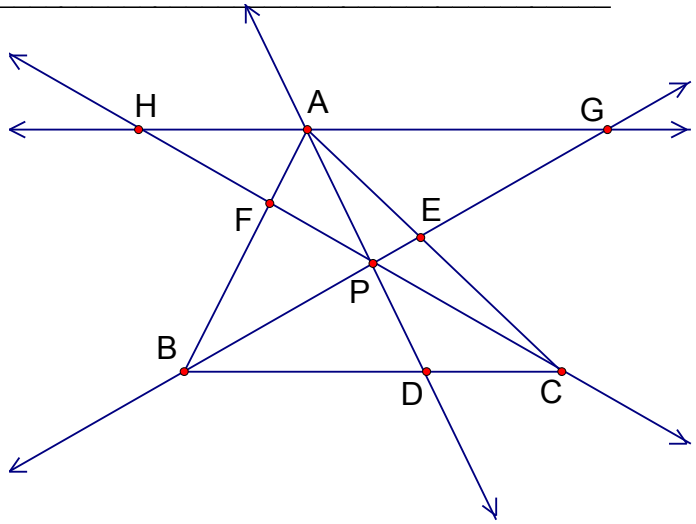
6. Is the following a valid proof in **Euclidean** geometry? If the proof is incorrect, indicate the errors. (Remember, you can give an incorrect proof for a correct theorem).



Theorem: Given ΔABC and an arbitrary point P, let the lines $\overleftrightarrow{AP}$, $\overleftrightarrow{BP}$, and $\overleftrightarrow{CP}$, intersect the opposite sides of the triangle at D, E, and F as shown. Then

$$FA \times DB \times EC = FB \times DC \times EA .$$

Proof: Draw a line through A parallel to $\overline{BC}$. Let $\overleftrightarrow{BP}$ intersect this line at G and let $\overleftrightarrow{CP}$ intersect the line at H. Then $\Delta APH \approx \Delta DPC$, from which we deduce $\frac{PD}{AP} = \frac{DC}{AH}$, and $\Delta APG \approx \Delta DPB$, from which we deduce $\frac{PD}{AP} = \frac{DB}{AG}$. We apply the transitive law to these two equations to obtain $\frac{DB}{AG} = \frac{DC}{AH}$, from which we get

$$\frac{DB}{DC} = \frac{AG}{AH} .$$

Now we also have $\Delta AFH \approx \Delta BFC$ and $\Delta AEG \approx \Delta CEB$ from which we deduce

$$\frac{FA}{FB} = \frac{AH}{BC} \text{ and } \frac{EC}{EA} = \frac{BC}{AG}$$

Now multiplying the three displayed equations together gives us $\frac{FA}{FB} \times \frac{DB}{DC} \times \frac{EC}{EA} = 1$, and the conclusion of our theorem follows immediately from this.

Note: Don't worry about what happens if P is outside the triangle. If there is an error, that isn't it. If there isn't an error, this proof would be valid for that case as well, though we would need to let D be the intersection with the *line* $\overleftrightarrow{BC}$, etc.

Name: _____

7. Are the following statements true in Euclidean geometry, hyperbolic geometry, both or neither?

    a. If two parallel lines are cut by a transversal, the alternate interior angles are equal.

    b. If two lines are cut by a transversal such that the corresponding angles are equal, then the lines are parallel.

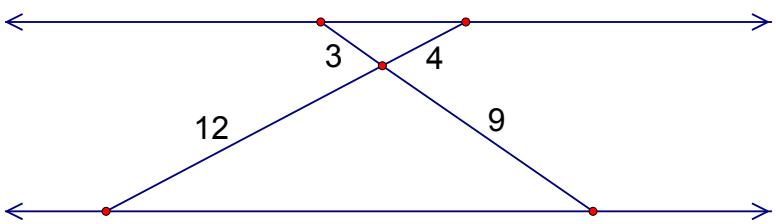
    c. The square on the hypotenuse of a right triangle is equal to the sum of the squares on the other two sides.

    d. The summit angles of a Saccheri quadrilateral are congruent (note that the definition of Saccheri quadrilateral makes sense for Euclidean geometry as well as hyperbolic).

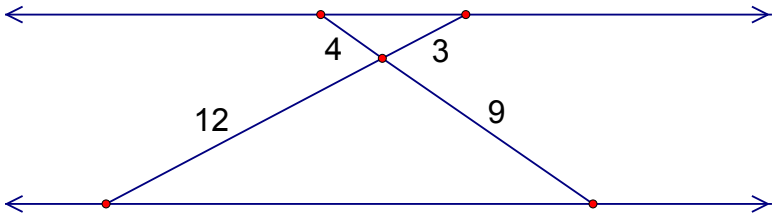
Name: _____

8. Answer the following questions in **Euclidean** geometry.

a. Are these lines, parallel, not parallel, or are you unable to tell? Justify your conclusions.



b. Are these lines, parallel, not parallel, or are you unable to tell? Justify your conclusions.



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9. **Explain how we know statements in geometry are true and how this differs from how we decide statements are true in other sciences**. Your explanation should include a discussion of the roles of both axiom systems and proofs, as well as a consideration of why we accept certain axiom systems. If you wish, you may take issue with the words "know" and/or "true" in the question.

Name: _____

Pledge:
On my honor, as a student, I have neither given nor received unauthorized aid on this examination: _____ _____

(signature) (date)