Math 9C Homework 7

Commonly Asked Questions

1. Solution to the in-class problem: Find the radius of convergence and the interval of convergence for the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n+1}.$$

To find the radius of convergence, we use the ratio test:

$$\lim_{n\to\infty} \left| \frac{\frac{(-1)^{n+1}x^{n+1}}{n+2}}{\frac{(-1)^n x^n}{n+1}} \right| = \lim_{n\to\infty} \frac{|x|^{n+1}(n+1)}{|x|^n(n+2)} = \lim_{n\to\infty} \frac{|x|(n+1)}{n+2} = |x|.$$

The ratio test tells us that this series converges for $|x| < 1$ and diverges for $|x| > 1$. Thus, the radius of convergence is 1.

To find the interval of convergence, we still need to check the endpoints, where $|x| = 1$. When $x = 1$, we get the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$$

which you can check converges by the alternating series test.

When $n = -1$, we get the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n(-1)^n}{n+1} = \sum_{n=0}^{\infty} \frac{(-1)^{2n}}{n+1} = \sum_{n=0}^{\infty} \frac{1}{n+1}$$

which you can check diverges by the limit comparison test, comparing with the harmonic series

$$\sum_{n=0}^{\infty} \frac{1}{n}.$$

Thus, the interval of convergence is $(-1, 1]$.

2. There was some difficulty with the definition of radius of convergence. Here is a precise definition:

Given a power series

$$\sum_{n=0}^{\infty} c_n (x-a)^n$$

its *radius of convergence* is the number R such that the series converges when $|x-a| < R$ and diverges when $|x-a| > R$.