

**Math 9C Homework 5**  
Commonly Asked Questions

1. Solution to the homework problem of the day:

$$\sum_{n=1}^{\infty} \frac{3}{n(n+3)}.$$

This series looks like it could be a telescoping series, so we find a partial fraction decomposition:

$$\frac{A}{n} + \frac{B}{n+3} = \frac{3}{n(n+3)},$$

which we solve by looking at

$$A(n+3) + Bn = 3.$$

Plugging in  $n = 0$  and  $n = -3$  gives  $A = 1$  and  $B = -1$ .

Thus, we now look at

$$\sum_{n=0}^{\infty} \left( \frac{1}{n} + \frac{1}{n+3} \right).$$

The first few partial sums are as follows:

$$s_1 = 1 - \frac{1}{4}$$

$$s_2 = \left(1 - \frac{1}{4}\right) + \left(\frac{1}{2} - \frac{1}{5}\right)$$

$$s_3 = \left(1 - \frac{1}{4}\right) + \left(\frac{1}{2} - \frac{1}{5}\right) + \left(\frac{1}{3} - \frac{1}{6}\right)$$

$$s_4 = \left(1 - \frac{1}{4}\right) + \left(\frac{1}{2} - \frac{1}{5}\right) + \left(\frac{1}{3} - \frac{1}{6}\right) + \left(\frac{1}{4} - \frac{1}{7}\right)$$

$$s_5 = \left(1 - \frac{1}{4}\right) + \left(\frac{1}{2} - \frac{1}{5}\right) + \left(\frac{1}{3} - \frac{1}{6}\right) + \left(\frac{1}{4} - \frac{1}{7}\right) + \left(\frac{1}{5} - \frac{1}{8}\right)$$

Continuing, we see that the terms begin to cancel out, except for the initial  $1$ ,  $\frac{1}{2}$ , and  $\frac{1}{3}$ . Or, notice that we always have

$$s_n = 1 + \frac{1}{2} + \frac{1}{3} - \frac{1}{n+1} - \frac{1}{n+2} + \frac{1}{n+3}$$

and

$$\lim_{n \rightarrow \infty} s_n = 1 + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}.$$

2. What if you get a limit of 0 or  $\infty$  when using the limit comparison test?

Then the test gives no result. You cannot tell in this case whether your series converges or diverges, and so you either need to compare to a different known series, or you need to use a different test.

3. Why are there so many different tests for convergence or divergence of series?

We need many different tests because they all have different conditions, and not all series can be tested in the same way.