Math 9C Homework 25

Commonly Asked Questions

1. Solution to the in-class problem: Solve the differential equation

$$y' + y = \sin(e^x).$$

This equation is linear with $P(x) = 1$ and $Q(x) = \sin(e^x)$. Thus, we begin by finding the integrating factor

$$I(x) = e^{\int P(x)dx} = e^{\int dx} = e^x.$$

Now, we multiply both sides of the differential equation by this integrating factor to get

$$e^x[y' + y] = e^x \sin(e^x).$$

The integrating factor has been chosen precisely so that the left-hand side of this equation is $[e^x y]'$. Thus we solve by taking

$$e^x y = \int e^x \sin(e^x)dx.$$

Using substitution with $u = e^x$, we get

$$e^x y = -\cos(e^x) + c$$

and solving for y we get

$$y = \frac{-\cos(e^x) + c}{e^x}.$$

2. Will we have to know trig identities?

The only one I think we've used much is $1+\tan^2(x) = \sec^2(x)$, showing up in arc length problems, so you should know that one. I won't expect you to know things like double-angle formulas that came up in the homework recently.

3. Will there be any real-life scenario problems on the exam?

Any of the kinds of problems we have seen are fair game. So, mixing problems, electric circuit differential equations, and population problems, for example, are all things you should know how to do.

4. What do we write if we test for convergence or divergence of a series and the test is inconclusive?

   That means that you need to try another test. Any series I give you can be determined for convergence or divergence using at least one of the methods we have used.

5. I'm still having trouble finding little k for population problems.

   This depends a bit on what kind of information you are given. The method I gave you (and then corrected the next day) concerns when you have a data table and need to find a good estimate. On the other hand, when you have a problem which says something like "the number tripled in one year" then you should plug in amounts for $t = 1$ and solve for k using the formula.

6. For the limit comparison test, how do you know what to compare to?

   Generally, you want to compare to something that you know easily whether it converges or diverges, usually a p-series or a geometric series. If the terms of your series look similar to one of these kinds of series but have, say, an extra number added in, then that is a good bet for making a comparison.

7. Will we have to draw direction fields?

   It is possible.

8. Do we have to remember all the tests for convergence and divergence for series?

   Yes.

9. In the example in the book in section 10.5, how do they get that

$$e^{x^3}\frac{dy}{dx} = 3x^2e^{x^3}y = \frac{d}{dx}(e^{x^3}y)?$$

   This is something that always happens when we solve linear differential equations. The whole point of the integrating factor (in this example, found to be e^{x^3}), is that if you multiply the left-hand side by it, you get the derivative of the integrating factor times y. Although it often looks strange, we have chosen the integrating factor precisely so that this will happen.

10. Do you have to memorize the electric circuit formula?

    You do not have to memorize the differential equation that we've used, but you should know what all the variables stand for (current, voltage, etc.).

11. Do we need to know other models for population growth?

    You do not need to memorize their solutions. You could be asked to solve for one, but you would do it from scratch in this case.

12. Will any formulas be given to us?

    As mentioned above, I would give you the differential equation for electric circuits, or in any population model other that natural growth or logistic, I'd give you the equation to use.

13. How do you know when to use partial fractions?

    You need to use it when you have a fraction whose denominator is the product of two polynomials. For example, when you have

    $$\frac{1}{x^2 - 4}$$

    you know that $x^2 - 4 = (x - 2)(x + 2)$. To make it easier to work with, we want to break it up into two fractions, one with denominator $x - 2$ and the other with denominator $x + 2$. Then it will be much easier to integrate, for example, since we can integrate the two simpler fractions. Partial fractions is the way that we determine the numerators of these two fractions.

14. When computing surface area, how do you know when to use x or y in place of $f(x)$?

    When rotating a curve about the x-axis, the integral for surface area always begins with $2\pi y$. If you are integrating with respect to y (i.e., if all the other variables in the integral are y and has a dy at the end), then you leave it as a y. If you are integrating with respect to x, then you want this part to be written in terms of x also, so you use the equation $y = f(x)$ (for some function $f(x)$) to write instead $2\pi f(x)$.

    If you are rotating a curve about the y-axis, then the situation is reversed: the integral begins with $2\pi x$. Now you leave it as x if you are integrating with respect to x, and replace it using $x = g(y)$ if you are integrating with respect to y, and then use $2\pi g(y)$.

15. Will we have a modeling question?

    If you mean a population model question, then yes, it is possible.

16. How do you find the power series for something like

$$f(x) = \frac{x^2}{(1+x)^3}?$$

    There are two methods: one is to do it using Taylor's formula, by taking lots of derivatives of this function and finding a pattern. But, this method can get messy, and you might recall that we did problems like this even before we learned about Taylor series. First, we rewrite as

$$f(x) = x^2 \frac{1}{(1+x)^3}$$

    and focus on this second piece. Notice that, up to some constant, it looks something like the second derivative of $\frac{1}{x+1}$, a function which looks like the sum of a geometric series. Thus, you can start with $\frac{1}{(1+x)^3}$ and integrate twice to get something looking similar to $\frac{1}{x+1}$, and then use geometric series. Given this power series, take the derivative twice to get back to the function that you want. Then remember to multiply by x^2 in each term.

17. Is there a guideline for which test to use for series?

    As I did before the first exam, I recommend looking at section 12.7 for some suggestions. Lots of practice helps, too.

18. How do you know when to use the exponential or logistic models?

    Usually, it will be given to you. The only exception is when looking at a data table and determining which is better. But even then, you generally find values for both models, and compare to the data to see which fits best.

19. When doing Euler's method, do you multiply the step size by $F(x_{n-1}, y_{n-1})$ each time?

Yes, to find y_n, you take y_{n-1} and add to it the step size h by $F(x_{n-1}, y_{n-1})$, where $F(x, y) = \frac{dy}{dx}$.

20. Do we have to memorize all equations for chapter 10?

    The things you should memorize from chapter 10 are the exponential and logistic models and their solutions, the formula for Euler's method, and the method for solving a linear differential equation.