Math 9C Homework 19

Commonly Asked Questions

1. Solution to the in-class problem: Show that the function

$$y = \frac{\ln(x) + c}{x}$$

is a solution of the differential equation

$$x^2 y' + xy = 1.$$

Applying the quotient rule, we get that

$$y' = \frac{\frac{1}{x}(x) - (\ln(x) + c)}{x^2} = \frac{1 - \ln(x) - c}{x^2}.$$

Plugging into the differential equation, we get

$$x^2\left(\frac{1 - \ln(x) - c}{x^2}\right) = x\left(\frac{\ln(x) + c}{x}\right) = 1$$

which simplifies to

$$1 - \ln(x) - c + \ln(x) + c = 1$$

which gives $1 = 1$. Thus, this function is a solution.

Find the solution satisfying $y(1) = 2$.

Here, we use this initial condition to find a specific value of c. Plugging in $x = 1$ and $y = 2$, we get

$$2 = \frac{\ln(1) + c}{1} = c.$$

Thus, $c = 2$, and we get the solution

$$y = \frac{\ln(x) + 2}{x}.$$

2. What is the purpose of the methods in this section? (question to be answered)

The purpose here is NOT to find a solution. The idea is that, even if we cannot find a solution, we can find good approximations for solutions. Direction fields give us an idea what the graph of a solution looks like. Euler's method gives an estimate for the the answer we'd get if we plugged in a specific value to a solution.