

Math 9C Homework 17
Commonly Asked Questions

1. Solution to the in-class problem: Set up but do not integrate the following surface area integrals:

- (a) The surface obtained by rotating the curve

$$y = xe^{-x} \quad 1 \leq x \leq 3$$

about the x -axis

First we use the product rule to take the derivative to get

$$\frac{dy}{dx} = -xe^{-x} + e^{-x} = e^{-x}(1 - x).$$

Then, squaring, we get

$$\left(\frac{dy}{dx}\right)^2 (e^{-x} - xe^{-x})^2 = e^{-2x}(1 - x)^2.$$

Plugging into the formula for the surface area when we rotate around the x -axis, we get

$$S = \int_1^3 2\pi xe^{-x} \sqrt{1 + e^{-2x}(1 - x)^2} dx.$$

- (b) The surface obtained by rotating the curve

$$x = \ln(y + 1)y \leq 1$$

about the x -axis.

Taking the derivative, we obtain

$$\frac{dx}{dy} = \frac{1}{y + 1},$$

and squaring it gives

$$\left(\frac{dx}{dy}\right)^2 = \frac{1}{(y + 1)^2}.$$

Plugging into the formula, we get

$$S = \int_0^1 2\pi y \sqrt{1 + \frac{1}{(y + 1)^2}} dy.$$