Math 9C Homework 15

Commonly Asked Questions

1. Solution to the in-class problem: Find the length of the curve given by $y = 1 + 6x^{\frac{3}{2}}$ where $0 \le x \le 1$.

   We first take the derivative to get
   $$\frac{dy}{dx} = 9x^{\frac{1}{2}}.$$
   Squaring it, we get
   $$\left(\frac{dy}{dx}\right)^2 = 81x.$$
   Plugging this into the arc length formula, we get
   $$L = \int_0^1 \sqrt{1 + 81x}dx.$$
   To solve this integral, we do a u-substitution with $u = 1+81x$, in which case $du = 81dx$. Thus, we get that the arc length integral is equal to
   $$\int_1^{82} \frac{1}{81}u^{\frac{1}{2}}du = \frac{1}{81}\left(\frac{2}{3}u^{\frac{3}{2}}\right)|_1^{82} = \frac{2}{243}\left(82^{\frac{3}{2}} - 1\right).$$
   This is your exact answer; if you want a decimal approximation, you can plug into your calculator to see that it is roughly 6.103.

2. What is the difference between the arc length formula and the arc length function?

   The arc length formula gives the length of a specific arc with specific endpoints a and b:
   $$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}dx.$$
   The arc length function, on the other hand, has only the first point, a, fixed, and the other endpoint can vary:
   $$s(x) = \int_a^x \sqrt{1 + \left(\frac{dy}{dt}\right)^2}dt.$$

Thus, if we change the endpoint b, we don't have to start over and do the formula again; we can use the function and just plug in different values for b into x and get the arc length.

3. What if you aren't given endpoints?

   In this case, the function is only defined in a certain interval, so you find the length of the whole curve, with endpoints given by where the function is defined.