

Math 9C Homework 12
Commonly Asked Questions

1. Solution to the in-class problem: Find $T_3(x)$ for $f(x) = \sin(x)$, $a = \frac{\pi}{6}$. How accurate is it when $0 \leq x \leq \frac{\pi}{3}$?

We need to find the first few terms of the Taylor series for $f(x) = \sin(x)$ centered at $a = \frac{\pi}{6}$, so we need to take up through the third derivative and evaluate at $\frac{\pi}{6}$. Doing so gives the following:

$$f(x) = \sin(x), f\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$f'(x) = \cos(x), f'\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$f''(x) = -\sin(x), f''\left(\frac{\pi}{6}\right) = -\frac{1}{2}$$

$$f^{(3)}(x) = -\cos(x), f^{(3)}\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}.$$

Putting this information into the Taylor formula, we get

$$T_3(x) = \frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right) - \frac{1}{2 \cdot 2!}\left(x - \frac{\pi}{6}\right)^2 - \frac{\sqrt{3}}{2 \cdot 3!}\left(x - \frac{\pi}{6}\right)^3$$

which simplifies to

$$T_3(x) = \frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right) - \frac{1}{4}\left(x - \frac{\pi}{6}\right)^2 - \frac{\sqrt{3}}{12}\left(x - \frac{\pi}{6}\right)^3.$$

To find the error, we look at $f^{(4)}(x) = \sin(x)$. For $0 \leq x \leq \frac{\pi}{3}$, the largest value of $|\sin(x)|$ is $|\sin(\frac{\pi}{3})| = \frac{\sqrt{3}}{2}$, and the largest value of $|x - \frac{\pi}{6}|$ is $\frac{\pi}{6}$. Thus, we get

$$|R_3(x)| \leq \frac{\sqrt{3}}{4!}\left(x - \frac{\pi}{6}\right)^4 \leq \frac{\sqrt{3}}{48}\left(\frac{\pi}{6}\right)^4 \approx .0027.$$