

Math 9C Homework 11
Commonly Asked Questions

1. Solution to (what should have been) the in-class problem: Find the Maclaurin series for the function $f(x) = e^{5x}$.

We know that the Maclaurin series for e^x is

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

Replacing x with $5x$ we get that

$$e^{5x} = \sum_{n=0}^{\infty} \frac{(5x)^n}{n!}.$$

2. What is meant by the notation

$$\binom{k}{n}?$$

This is just a short notation for

$$\frac{k(k-1)\cdots(k-n+1)}{n!}.$$

3. What is the binomial series good for?

In this section, we are finding power series representations for functions by using Theorem 5. Using this method, we write the power series using derivatives of the function we are trying to represent. While this isn't generally hard, it can still be nice to have a shortcut!

The binomial series tell us that any time we have a function of the form $f(x) = (1+x)^k$, for any real number k , we don't have to take all these derivatives and find the Maclaurin series from scratch. We can just plug in a value of k into the formula for the binomial series.

4. We know the binomial series converges for $|x| < 1$, but what about the endpoints?

This will depend on what the value of k is. You'd have to check it by hand for a given example.

5. Why would we want to multiply and divide power series?

As with the binomial series, we can always find the Taylor series from scratch. But, if the function is a product (or quotient) of a function that we already know, it is nice to have a shortcut rather than having to work out all the derivatives again (this time using the product or quotient rule).

6. If we can do multiplication and division of power series, can we do them for other series?

Yes and no. If you multiply two series, you have to multiply every term of one series with every term of the other. (Think of high school algebra when you took $(a + b)(c + d)$. You have to multiply a times c and a times d , and then b times c and b times d . Here you do the same, but with infinitely many terms.) You can always do this for any series. You just have to remember that $(\sum a_n)(\sum b_n)$ is NOT the same thing as $\sum a_nb_n$. (This would be like saying that $(a + b)(c + d)$ is the same as $ac + bd$, which it is not - you are missing two terms!)

The reason it comes up in this section is because we often find products of functions, and we want to know their Taylor series. From there, see the answer to the previous question.