

Math 9C Homework 10
Commonly Asked Questions

1. Solution to the in-class problem: Evaluate the indefinite integral

$$\int \frac{t}{1-t^8} dt$$

as a power series and find its radius of convergence.

The function

$$f(t) = \frac{t}{1-t^8}$$

looks like the sum of a geometric series with $a = t$ and $r = t^8$. Thus, we can write it as a power series

$$\sum_{n=0}^{\infty} t(t^8)^n = \sum_{n=0}^{\infty} t^{8n+1}.$$

Taking the integral of this power series, we get

$$\int \sum_{n=0}^{\infty} t^{8n+1} dt = c + \sum_{n=0}^{\infty} \frac{t^{8n+2}}{8n+2}.$$

This series converges when $|t^8| < 1$, so when $|t| < 1$; thus the radius of convergence is 1.

2. Are there any functions that don't have a power series expansion? How do we deal with them?

For the purposes of this class, we can find power series expansions for all functions, but these power series only represent those functions in the interval of convergence. Outside of that interval, they may not converge, i.e., they don't represent that function for those values of x . In this class, we are mainly concerned with functions that do have power series expansions, at least in some interval.

3. Can Taylor's inequality be explained visually?

Well, yes, but difficult to explain on paper. Come and ask me or your TA if you want a better explanation!

4. Do you always have to use the ratio test to find the radius of convergence?

No, but it is often the most helpful.

5. Why are Taylor and Maclaurin series helpful?

The idea is that we take functions that can be very complicated, and make them look like (infinite) polynomials. Polynomials are nice for many reasons; in particular it is very easy to find their derivatives and integrals.

6. Why is $0! = 1$?

While there are more sophisticated answers (those of you in Edward's TA section should ask him if you want to know!), for the purposes of this class you can think of it as a convention that makes familiar formulas for factorials work nicely. For example, if you look at binomial coefficients for integers, we know that

$$\binom{k}{n} = \binom{k}{k-n}.$$

(You can use the formula

$$\binom{k}{n} = \frac{k!}{n!(k-n)!}$$

to see this.) But if we want

$$\binom{k}{k}$$

we want the answer to be 1 (number of ways to choose k items out of a possible k), but computing from the formula gives a $0!$ in the denominator. Thus, we want $0!$ to be 1.

7. What is the difference between a Taylor series and a Maclaurin series?

A Taylor series for a function x is the power series

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n.$$

We use the special case where $a = 0$ so often that it has its own name: the Maclaurin series.

8. Will we have to memorize the Taylor series for e^x or trig functions?

I strongly recommend you know the ones in Table 1. You either need to have them memorized, or be able to reconstruct them.