

Math 9C Homework 1

Commonly Asked Questions

1. What does a recursive formula mean?

The idea behind a recursive formula is that you give a beginning term (or sometimes more than one term, as in the Fibonacci sequence - however many you need) and then define all the rest of the terms from them. So, if you want to know the n th term a_n , you need to compute all the previous terms a_1, a_2, \dots, a_{n-1} and then use the recursive formula to find it.

In contrast, when you have a direct formula, you can compute a_n directly (hence the name) without having to know every single term that comes before it. So, usually it is nicer to have a direct formula, and in the problems on WebAssign, they are looking for a direct formula. But, sometimes a recursive formula is easier to spot, and it can be used to help you find a direct formula.

For example, in the homework exercise, where you had to find different formulas for the sequence 2, 7, 12, 17, \dots , you might first notice that you just add 5 each time to get the next term. Hence, the recursive formula $a_{n+1} = a_n + 5$. Note that you have to specify the first term, though! So, you need to include $a_1 = 2$. Otherwise, you wouldn't know where to begin the sequence! Now, you know that you are adding 5 each time, so when you look at the $n + 1$ st term, you have added 5 to your original term n times. But, repeated addition is just multiplication, so taking $5n$ is a good start to a direct formula. It isn't quite right, however, as it is, since it would have a starting term of 5, and we need the first term to be 2. So, we subtract 3, to get $a_n = 5n - 3$. You can check the other terms to see that this direct formula does in fact work.

2. What is the second definition of a limit of a sequence mean? What is the ε ?

Mathematicians have to be really precise. It isn't precise enough to say that the terms of a sequence "get close to L " so they want a really specific way to say it. What does close mean? Well, it means that if we look at the terms of the sequence, which are just numbers, we want the distance between each term and the limit L to get as small as you

like. (To visualize, you might want to think about points on a number line.) So, we say, pick a very small number, any number you like, and call it ε . Whatever ε is, we want the terms to have distance smaller than it from the number L , at least if we go far enough in the sequence. How far is far enough? That will depend on the sequence and on ε , but even if it is very large, we can always find a number N such that all terms after a_N (i.e., a_n for all $n \geq N$) will be closer than ε from L . Writing in symbolic notation, we get $|a_n - L| < \varepsilon$.

We won't use this definition much, so if you understand the first one, that is great. But, it is good to understand why mathematicians need this second version - especially for those of you who will keep studying math!

3. Is there a method we can use to find formulas for sequences?

No, not really. Formulas can be very different from one another. But, doing lots of examples will help. Look for patterns - are you adding or multiplying by the same number each time? Are the signs staying the same or alternating? With practice, you'll find this easier.