Math 570 - Practice Exam Solutions

1. Using the form $x^3 + ax^2 + bx + c = 0$, we have $a = 6$, $b = 21$, and $c = 10$. Thus, the substitution $x = y - \frac{a}{3}$ in this case will be $x = y - 2$. Plugging in and simplifying yields:

$$\begin{aligned}(y-2)^3 + 6(y-2)^2 + 21(y-2) + 10 &= 0\\ (y-2)(y^2 - 4y + 4) + 6(y^2 - 4y + 4) + 21y - 42 + 10 &= 0\\ y^3 - 6y^2 + 12y - 8 + 6y^2 - 24y + 24 + 21y - 32 &= 0\\ y^3 + 9y - 16 = 0 \text{ or } y^3 &= 9y + 16.\end{aligned}$$

Since this equation is of the form $y^3 + py = q$ (with $p = 9$ and $q = 16$), we use the first of the formulas given by Cardano:

$$\begin{aligned}y &= \sqrt[3]{\sqrt{\frac{16^2}{4} + \frac{9^3}{27}} + 8} - \sqrt[3]{\sqrt{\frac{16^2}{4} + \frac{9^3}{27}} - 8}\\ &= \sqrt[3]{\sqrt{64+27} + 8} - \sqrt[3]{\sqrt{64+27} - 8}\\ &= \sqrt[3]{\sqrt{91} + 8} - \sqrt[3]{\sqrt{91} - 8}\end{aligned}$$

Then, plugging in for y, we get

$$x = \sqrt[3]{\sqrt{91} + 8} - \sqrt[3]{\sqrt{91} - 8} - 2.$$

2. (a) Let $x = 32.7 \times 6.93$. Using the rules for logarithms and the table, we can obtain

$$\begin{aligned}\log(32.7 \times 6.93) &= \log(3.27 \times 10 \times 6.93)\\ &= \log(3.27) + \log(10) + \log(6.93)\\ &\approx .5145 + 1 + .8407\\ &= 1 + 1 + .3552\\ &\approx \log(10) + \log(10) + \log(2.27)\\ &= \log(10 \times 10 \times 2.27)\\ &= \log(227).\end{aligned}$$

Thus, $x \approx 227$.

(b) Let $x = \sqrt[3]{27.6}$ as before, and proceed as previously.

$$\begin{aligned}
\log(\sqrt[3]{27.6}) &= \frac{1}{3}\log(2.76 \times 10) \\
&= \frac{1}{3}\log(2.76) + \frac{1}{3}\log(10) \\
&\approx \frac{1}{3}(.4409) + \frac{1}{3} \\
&= .1469\overline{6} + \frac{1}{3} \\
&= .4803 \\
&\approx \log(3.02).
\end{aligned}$$

Thus, $x \approx 3.02$.

3. Using the Euclidean algorithm and long division, you can get

$$gcd(399, 525) = gcd(126, 399) = \gcd(21, 126) = 21.$$

4. Considering this chart as a grid, the coefficients of $2x^2+5x-1$ are found along the top row, and the coefficients of x^2-2x+3 are along the rightmost column. The boxes directly under and two the left of these are obtained via multiplication. Then, these values are added diagonally (upper right to lower left) to obtain the values running down the leftmost column and into the bottom row, which are the coefficients of $2x^4 + x^3 - 5x^2 + 17x - 3$. The reason it works is because you are essentially multiplying term-by-term and then grouping together the coefficients of the same-degree terms when you add diagonally.

   This technique is similar to that of the various forms of "chessboard multiplication found in the *Treviso Arithmetic* and would generalize to higher-order polynomials, assuming that everything was aligned correctly.

5. Euler was a mathematician of the eighteenth century. He is well-known for his *Introductio in Analysin Infinitorum* which systematized what was known of calculus in his time. He also did work in number theory and began the field of graph theory. (You don't have to explain the pronunciation of his name.)

6. Ptolemy was a Greek mathematician in the second century AD. He did work in geometry and found an approximation for the circumference of the earth which was accepted for several hundred years.

7. Wallis was a British mathematician in the seventeenth century who computed areas of figures using the notions of infinitesimals and infinite series. His work was influential for Newton and is considered part of the beginning of calculus.

8. Descartes was a French mathematician in the seventeenth century who brought algebra and geometry together in analytic geometry. His ideas led to our use of cartesian coordinates and were also influential in the development of calculus.

9. (We didn't cover this.)

10. Negative numbers and complex numbers were introduced in Cardano's *Ars Magna*.

11. The passage is from Descartes' *Geometry* and describes his "circle method" for finding normal and tangent lines to a point on a curve.

12. Between Diophantus and Cardano, algebra was developed more fully with the Arab mathematicians. In particular, the Hindi-Arabic numerals were adopted in Europe and numbers were able to be manipulated symbolically in a way they had not previously been.

    Note: These last two questions ask for one or two paragraphs, which I have not given. I've just given a brief prompt to help if they stump you.