

Name: \_\_\_\_\_

The following exam is worth 150 points. You are permitted to use a calculator, except where otherwise indicated. **You must show all work to receive full credit.** You have 2 hours to complete this exam. You may leave when you are finished. Good luck!

**You are taking this exam under the Honor Code.**

[Comic goes here.]

1. Evaluate the following definite integrals:

(a)  $\int_0^1 3xe^x dx$

(b)  $\int_0^2 x\sqrt{x^2 + 4}dx$

2. Find the area enclosed by the graphs of  $y = -x^2 + 4$  and  $y = -x + 2$ .

3. Use the Riemann sum method and left endpoints to find the area under the graph of  $y = 3x$  on the interval  $[0,1]$ .



6. Find the equation of the tangent line to  $f(x) = e^x + x$  at  $x = 0$ .

7. Find the following indefinite integral:

$$\int \frac{dx}{x^2 - 3x - 4}$$

8. Brian decides that he should save money now to buy football tickets 3 years from now. If a season pass is expected to cost \$120 in 2005, how much does he have to invest at 1.5% interest, compounded continuously, to have enough money then?

9. Chris and Jordan discover a strange radioactive element lurking under the bleachers in the Joyce Center. After 5 days it has decayed to 20% of its original amount. What is the half-life of the element?

10. While working at Rolfs, Kara has been asked to determine how many aerobics classes should be offered in the fall. If 12 classes are offered, the profit earned is \$200. At that point, the marginal profit is \$2.50. Using linear approximation, estimate the profit earned if 15 aerobics classes are offered.

11. Consider the following function:

$$f(x) = \frac{x + 1}{x - 1}$$

- (a) What is the domain of  $f(x)$ ?

(b) Where is  $f(x) = 0$ , if anywhere?

(c) Does  $f(x)$  have any vertical asymptotes? If so, where?

(d) Does  $f(x)$  have any horizontal asymptotes? If so, where?

(e) On what intervals is  $f(x)$  increasing and decreasing?

(f) Does  $f(x)$  have any critical points? If so, what are they?

(g) On what intervals is  $f(x)$  concave up and concave down?

(h) Does  $f(x)$  have any inflection points? If so, what are they?

(i) Sketch the graph of  $f(x)$ . Do not use a calculator.

12. While studying in France, Cory decides to take the train through the Chunnel to England. If the velocity of the train after  $t$  minutes is given by  $v(t) = t^2 + t$ , how far will the train have gone from its starting position after 7 minutes?

13. Jenn decides to build a lacrosse field in her backyard. The dimensions of the field are 80 yards by 40 yards, and she wants a spectator area around it. If she has 400 yards of fencing available to make a rectangle around the spectator area, what dimensions will maximize the spectator area minus the playing field?

14. Consider the function  $f(x)$  graphed below.

(a) At what  $x$  values does the limit not exist?

(b) At what  $x$  values is  $f(x)$  not continuous?

(c) At what  $x$  values is  $f(x)$  not differentiable?

For parts (d)-(g), consider **only** the interval  $[4, 11]$ .

(d) Where is  $f'(x) > 0$ ? Where is  $f'(x) < 0$ ?

(e) Where is  $f''(x) > 0$ ? Where is  $f''(x) < 0$ ?

(f) Where does  $f(x)$  have local maxima and minima?

(g) Where does  $f(x)$  have a global maximum and minimum?

15. After taking the final exam, Paris is so excited to be done with calculus that he throws his book off the roof the math building. If the building is 48 feet high and the height of the book  $t$  seconds after being thrown is given by  $s(t) = -16t^2 + 32t + 48$ , what is the velocity of the book when it hits the ground?