Abstracts

Vyjayanthi Chari, UCR

Posets of Partitions, Row Shuffles and Schur positivity

A well-known integral basis for the ring of symmetric functions in $n$ variables is the set of Schur functions $s_{\lambda}$, where $\lambda$ varies over partitions with at most $n$ parts. A symmetric function is said to be Schur positive if it can be written as non-negative integer linear combination of Schur functions. Fomin, Fulton, Li and Poon defined the notion of a row shuffle of a pair of partitions: given two partitions $\lambda$ and $\mu$, the row shuffle is a pair $(\nu_1, \nu_2)$ partitions obtained by shuffling the rows of the Young diagrams of $\lambda$ and $\mu$. They conjectured (now a theorem due to T. Lam, A. Postnikov and P. Pylyavskyy, that the difference $s_{\nu_1}s_{\nu_2} - s_{\lambda} - s_{\mu}$ is Schur positive. In terms of representation theory of the general linear algebra $\mathfrak{gl}_n$ this question amounts to asking when the following to holds:

$$\dim \text{Hom}_g(V, V(\lambda) \otimes V(\mu)) \leq \dim \text{Hom}_g(V, V(\nu_1) \otimes V(\nu_2)),$$

where $V$ is an arbitrary irreducible representation of $\mathfrak{gl}_n$ and $V(\lambda)$ is the irreducible finite-dimensional $g$-module whose character is $s_{\lambda}$. In this talk we shall discuss a partial order on pairs (more generally $k$-tuples) of partitions which "add" up to the same partition. We shall see that the maximal element in this partial order coincides with the row shuffle of partitions defined by Fomin, Fulton, Li and Poon. In joint work with Fourier and Sagaki, we conjecture that that the inequality discussed above holds along the partial order. We shall discuss the cases when the conjecture is known to be true.

Cynthia V. Flores, UCSB

On a decay property of solutions to the Benjamin-Ono equation

In this talk we investigate unique continuation properties of solutions to the initial value problem associated to the Benjamin-Ono equation in weighted Sobolev spaces $Z_{s,r} = H^s(\mathbb{R}) \cap L^2(|x|^{2r}dx)$ for $s \in \mathbb{R}$, and $s \geq 1, s \geq r$.

Johanna Hennig, UCSD

Locally finite Lie algebras in positive characteristic

For finite dimensional Lie algebras, there is the well-known Ado’s theorem: Every finite dimensional Lie algebra embeds into a finite dimensional associative algebra. Bahturin, Baranov, and Zalesski proved an infinite dimensional version of Ado’s theorem for a simple, locally finite Lie algebra $L$ over a field of characteristic zero: $L$ embeds into a locally finite associative algebra if and only if $L$ is isomorphic to the commutator of skew-symmetric elements of a locally finite, associative algebra with involution. We extend this result to fields of positive characteristic–we provide two structure theorems which reduce to Bahturin, Baranov, Zalesski’s result in characteristic zero and also generalize classical structure theorems for finite dimensional Lie algebras in characteristic $p$.

Maree Jaramillo, UCSB

On Fundamental Groups and Smooth Metric Measure Spaces

The Bakry-Emery Ricci tensor is a natural extension of Ricci curvature on smooth metric measure spaces. Since topological and geometric information can be obtained for manifolds with Ricci curvature bounded
from below, it is natural to ask if the same information holds true for smooth metric measure spaces with Bakry-Emery Ricci tensor bounded from below. Using Guofang Wei and Will Wylie’s comparison theorems and an extension of Kevin Brighton’s gradient estimate on smooth metric measure spaces, we extend the Almost Splitting Theorem of Cheeger-Colding to the smooth metric space setting. Using this Almost Splitting theorem, we show that the fundamental group of the smooth metric measure space with a lower bound on volume has almost abelian fundamental group. We also show that the number of generators of the fundamental group of a smooth metric measure space with Bakry-Emery Ricci tensor bounded from below is uniformly bounded. The results on the fundamental group are extensions of theorems which hold for Riemannian manifolds with Ricci curvature bounded from below.

Gizem Karaali, Pomona College

Supercharacters and Their Superpowers

In this talk I will introduce supercharacter theory, a generalization of character theory, which was developed originally by Carlos Andre and then picked up and studied more extensively by Persi Diaconis and I. M. Isaacs. This new development in finite group representation theory has spurred much exciting work, which will be briefly overviewed in the talk. I will begin at the (very) beginning, so no prior experience in supercharacter theory is required.

Helen Parks, UCSD

Using math in industry: Implementing a smarter graphics algorithm

I will describe a computer graphics algorithm that leverages mathematical ideas to improve graphics performance. I implemented a version of this algorithm as part of my work at a summer internship with the Boeing Applied Math Group. The algorithm uses subdividable, linear function enclosures (slefes). Jorg Peters developed slefes as a way to tightly bound spline curves using piecewise linear curves. Splines are one of the most common ways to represent geometry in computer science, making spline graphics rendering a common problem. Peters’ iPASS algorithm leverages slefe bounds and their theory to ensure pixel-accurate rendering of spline geometries.

Jacquelyn Rischke, UCI

Regularization of languages by learners: a mathematical framework

E.L. Newport and colleagues have demonstrated that both children and adults have some ability to process inconsistent linguistic input and “improve” it by making it more consistent. We create a learning algorithm of the reinforcement-learning type, which exhibits patterns reported by Hudson Kam and Newport (2009) and suggests a way to explain them. In order to capture the differences between children’s and adults’ learning patterns, we need to introduce a certain asymmetry in the learning algorithm. Namely, we have to assume that the reaction of the learners differs depending on whether or not the source’s input coincides with the learner’s internal hypothesis. We interpret this result in the context of a different reaction of children and adults to positive and negative evidence. We propose that a possible mechanism that contributes to the children’s ability to regularize an inconsistent input is related to their heightened sensitivity to positive evidence rather than the (implicit) negative evidence. In our model, regularization comes naturally as a consequence of a stronger reaction of the children to evidence supporting their preferred hypothesis. The adults’ ability to adequately process implicit negative evidence prevents them from regularizing the inconsistent input.
Amanda Ruiz, Harvey Mudd College

Shi arrangements, parking functions, and mixed graphs

The Shi arrangement $Shi(n)$ is the set of all hyperplanes in $\mathbb{R}^n$ of the form $x_j - x_k = 0$ or 1 for $1 \leq j < k \leq n$. Shi observed in 1986 that the number of regions (i.e., connected components of the complement) of this arrangement is $(n + 1)^{(n-1)}$. An unrelated combinatorial concept is that of a parking function, i.e., a sequence $(x_1, x_2, ..., x_n)$ of positive integers that, when rearranged from smallest to largest, satisfies $x_k \leq k$. It turns out that the number of parking functions of length $n$ also equals $(n + 1)^{(n-1)}$, a result due to Konheim and Weiss from 1966. (We will explain the reason for the name "parking function"). So a natural problem consists of finding a bijection between $Shi(n)$ and the parking functions of length $n$. Stanley and Pak (1996) and Athanasiadis and Linusson (1999) gave such (quite different) bijections. We will present a new (and arguably most natural) bijection, which takes a scenic route through certain mixed graphs. (We will not assume any knowledge of hyperplane arrangements or graph theory.) This is joint work with Matthias Beck, Ana Berrizbeitia, Michael Dairyko, Claudia Rodriguez, and Schuyler Veeneman.

Lisa Schneider and Peri Shereen, UCR

Modules with Demazure Flags and Character Formulae: Part 1 and Part 2

In this talk, we discuss a family of finite–dimensional graded representations of the current algebra of $\mathfrak{sl}_2$ which are indexed by partitions. We show that for $\ell$ sufficiently large, these representations admit a filtration by submodule where the successive quotients are Demazure modules which occur in a level $\ell$ integrable module for $A_1$. We associate to each partition and to each $\ell$ an edge–labeled directed graph which allows us to describe in a combinatorial way the graded multiplicity of a given level $\ell$–Demazure module in the filtration. In the special case of the partition $1^s$ and $\ell = 2$, we give a closed formula for the graded multiplicity of level two Demazure modules in a level one Demazure module. As an application, we use our result along with the results of K. Naoi and Lenart et al, to give the character of a $g$–stable level one Demazure module associated to $B_n^1$ as an explicit combination of suitably specialized Macdonald polynomials. In the case of $\mathfrak{sl}_2$, we also study the filtration of the level two Demazure module by level three Demazure modules and compute the numerical filtration multiplicities and show that the graded multiplicites are related to (variants) of partial theta series.

Tatiana V. Tatarinova, USC

Reaching the Holy Grail of Biogeography – from Genome to Home Village

The search for a biogeographical method that utilizes biological information to predict one’s place of origin has occupied scientists for millennia. Modern biogeographical algorithms achieve an accuracy of 700 km in Europe but are highly inaccurate elsewhere, particularly in Southeast Asia and Oceania. Here, we present the Geographic Population Structure (GPS) algorithm that accurately infers the biogeography of worldwide individuals down to their village of origin. GPS’s accuracy is demonstrated on three datasets: worldwide populations, Southeast Asians and Oceanians, and Sardinians (Italy) using 40,000-130,000 GenoChip markers. GPS correctly placed 80% of worldwide individuals within their country of origin with an accuracy of 87% for Asians and Oceanians. Applied to over 200 Sardinians villagers of both sexes, GPS placed a quarter of them within their villages and most of the remaining within 50 km of their villages, allowing us to identify the demographic processes that shaped the Sardinian society. The accuracy and power of GPS has important ramifications for genetic ancestry testing, forensic and medical sciences, and genetic privacy.
Cindy Tsang, UCSB

**Galois module structure of abelian extensions**

Let $K$ be a number field with ring of integers $\mathcal{O}$ and $G$ an abelian group. Given a Galois $G$-extension $L/K$, its ring of integers $\mathcal{O}_L$ is an $\mathcal{O}G$-module. I will discuss how its module structure (global and local freeness) is related to the ramification of $L/K$. Furthermore, when it is locally free, it defines a class in the locally free class group $\text{Cl}(\mathcal{O}G)$ of $\mathcal{O}G$. One can show that the classes realizable by these $\mathcal{O}_L$ form a group. Finally, I will briefly talk about how one can generalize all of these to the square root of the inverse different $A_{L/K}$.

Katie Walsh, UCSD and USD

**Patterns and Stability in the Coefficients of the Colored Jones Polynomial**

The colored Jones polynomial assigns to each knot a sequence of Laurent polynomials. This talk will focus on the patterns in the coefficients of these polynomials. We will discuss the stabilization and higher-order stabilization of the coefficients, specifically discussing what the second $N$ coefficients of the $N$th colored Jones polynomial of certain knots stabilize to. We will also look at patterns in the middle coefficients.

Samantha Xu, UCLA

**Invariant Measures and Hamiltonian PDEs**

We explore the notion of an appropriate measure on the phase space of Hamiltonian systems, beginning with Liouville’s theorem for Hamiltonian ODEs. We then move to the infinite dimensional setting, with Hamiltonian PDEs, and discuss current methods of analysis and results.