

**MATH 145B INTRODUCTION TO TOPOLOGY, HOMEWORK
EXERCISES DUE FRIDAY, MAY 22**

(1) Show that the following definitions for $\mathbb{R}P^n$ all give homeomorphic spaces:

(a) Define a partition of S^n whose sets are pairs

$$\{(x_1, \dots, x_{n+1}), (-x_1, \dots, -x_{n+1})\}$$

and take its identification space.

(b) Define a partition of $\mathbb{R}^{n+1} \setminus \{0\}$ whose sets are

$$\{(cx_1, \dots, cx_{n+1}) \mid (x_1, \dots, x_{n+1}) \in \mathbb{R}^{n+1} \setminus \{0\}, c \in \mathbb{R} \setminus \{0\}\}$$

and take the resulting identification space.

(c) Define a partition of B^n whose sets are of either of the form

$$\{s, s\}, s \in S^{n-1}$$

or sets consisting of a single point in $B^n \setminus S^{n-1}$, and take the resulting identification space.