MATH 145B INTRODUCTION TO TOPOLOGY, HOMEWORK EXERCISES DUE FRIDAY, MAY 15

(1) Prove that if $f\colon X \to Y$ is a continuous and onto closed map, then f is an identification map.

(2) Show that the following definitions for $\mathbb{R}P^n$ all give homeomorphic spaces:

  (a) Define a partition of S^n whose sets are pairs

$$\{(x_1, \ldots x_{n+1}), (-x_1, \ldots, -x_{n+1})\}$$

  and take its identification space.

  (b) Define a partition of $\mathbb{R}^{n+1}\backslash\{0\}$ whose sets are

$$\{(cx_1, \ldots, cx_{n+1}) \mid (x_1, \ldots, x_{n+1}) \in \mathbb{R}^{n+1}\backslash\{0\}, c \in \mathbb{R}\backslash\{0\}\}$$

  and take the resulting identification space.

  (c) Define a partition of B^n whose sets are of either of the form

$$\{s, s\}, s \in S^{n-1}$$

  or sets consisting of a single point in $B^n\backslash S^{n-1}$, and take the resulting identification space.