IAMIS Summer Workshop on:

“The Applications of Fractal Geometry & Dynamical Systems Theory to Biology & Physics”

June 19th-22nd

CHASS Interdisciplinary Building North-Room 1002
University of California, Riverside

Organized by:
Michel Lapidus, Director of IAMIS; Professor, Department of Mathematics, UC Riverside
Richard Niemeyer, Deputy Director of IAMIS
Title: Spatial Scale and Population Dynamics in Advective Media.
Speaker: Kurt Anderson, University of California-Riverside
Abstract:

I will review recent research on mathematical models of populations that disperse in media with net unidirectional flow. Examples include drifting invertebrates in rivers and streams, marine organisms whose larvae are dispersed in local longshore currents, plants with wind or waterborne seeds, and gut microorganisms. I will focus on theory relating to three issues: conditions for population persistence, biotic responses to abiotic forcing, and stability. For both all issues, I will identify key length scales that impact qualitative dynamics. Population persistence in an idealized, spatially homogeneous, infinitely long system is commonly guaranteed if invasion waves can advance upstream. Demographic and dispersal characteristics of organisms in finite systems determine the “critical domain size”, i.e. the minimum system size for population viability. Spatial heterogeneity introduces a number of scenarios where population persistence involves source-sink dynamics. Interpretation of many aspects of steady state and transient responses to environmental forcing involves the “response length”, a measure of the distance over which the impact of a point-source disturbance is felt. Other lengths scales emerge when the system is unstable. The recent advances have implications for future theoretical, experimental and field work, along with policy development in the areas of conservation biology and environmental assessments. I will cover recent results for two emerging issues: application of metric graphs to river networks, and integration of hydrology with population dynamics.

Title: Multiscale Modularity in Brain Systems
Speaker: Danielle Basset, University of California-Santa Barbara
Abstract:

The physical and mathematical study of complex systems facilitates important insights into real-world phenomena. Recent developments in statistical mechanics and network theory together provide a principled framework in which to characterize the organization of such systems composed of many interacting parts. The human brain can be viewed as a model complex system, the study of which sits at the interdisciplinary boundary between statistical physics, applied mathematics, and neuroscience. Here we use a combination of mathematical modeling and time series analysis to characterize the organization of the human brain and to predict its behavior, facilitating a direct feedback loop between theory and experiment. We do so by constructing structural and functional brain networks using data from non-invasive neuroimaging techniques. These networks display a modular organization over both spatial and temporal scales that can be linked to cognitive performance. We explore the potential significance of this nontrivial structure in relation to both adaptive mechanisms and natural selection.
Title: Using the Structure of Inhibitory Networks to Unravel Mechanisms of Spatiotemporal Patterning  
Speaker: Maxim Bazhenov, University of California-Riverside  
Abstract:  

We established a relationship between an important structural property of an inhibitory network, its colorings, and the dynamics it constrains. Using a model of the insect antennal lobe we show that our description allows the explicit identification of the groups of inhibitory interneurons that switch, during odor stimulation, between activity and quiescence. This description optimally matches the perspective of the downstream neurons looking for synchrony in ensembles of pre–synaptic cells.

Title: A Nature-Inspired Approach to the Engineering of Efficient Catalytic Systems  
Speaker: Marc-Olivier Coppens, Rensselaer Polytechnic  
Abstract:  

Mechanisms used by biological systems to solve fundamental problems – such as those related to scalability, robustness and resource efficiency – could be applied to help designing solutions to similar challenges encountered in chemical production, energy and environmental technology. We call this methodology Nature-Inspired Chemical Engineering (NICE). NICE shows great potential to innovate reactor, catalysis and separation technology, when using a fundamentally rooted, mechanistic approach that is adapted to comply with the specific context of chemical engineering processes, rather than mimicry. In this presentation, we focus on the important role that fractal geometry and dynamics play in NICE.

Biology makes ample use of hierarchical networks to bridge scales and minimize transport limitations (of material, energy or information), leading to broadly scalable solutions that are robust, highly efficient, or both. Examples are trees, lungs and the vascular network. Such networks typically cross over from fractal on macro/mesoscopic scales to uniform at microscopic scales. The favorable properties resulting from these networks inspire the design of artificial fluid distribution and collection systems for chemical reactors, and porous catalyst architectures with optimal pore channel distributions that reduce diffusion limitations and mitigate the effects of catalyst deactivation. Applications to multiphase reactor engineering, heterogeneous catalysis, and fuel cells are discussed.

Nature also uses dynamics in various ways to create synergistically structured, complex, adaptable organizations from simple components. Examples are versatile bacterial communities, which can adopt fractal organizations. While common in nature, the mechanisms leading to these adaptive communities are only sporadically, purposefully applied in technology. Adaptive catalytic materials could be designed, inspired by self-assembly mechanisms.
Title: Non-Gaussian Dynamics, Nonlocal Operators and Applications
Speaker: Jinqiao Duan, IPAM, UCLA, Los Angeles; Illinois Institute of Technology, Chicago
Abstract:

Dynamical systems arising in biological, physical and chemical sciences are often subject to random influences, which are also known as “noise”. Stochastic differential equations are appropriate models for some of these systems. The noise in these stochastic differential equations may be Gaussian or non-Gaussian in nature. Non-Gaussianity of the noise manifests as nonlocality at a macroscopic level. In addition, randomness may have delicate, or even profound, impact on the overall evolution of dynamical systems. The speaker will present an overview of some available theoretical and numerical techniques for analyzing stochastic dynamical systems, including escape probability, mean exit time, invariant manifolds, bifurcation and quantifying the impact of uncertainty. The differences in dynamics under Gaussian and non-Gaussian noises are highlighted, in the context of a tumor growth system.

Title: SLE, KPZ and Liouville Quantum Gravity
Speaker: Bertrand Duplantier, CEA, Centre de Physique Theorique, France
Abstract:

When two boundary arcs of a Liouville quantum gravity random surface are conformally welded to each other (in a boundary quantum-length-preserving way) the resulting interface is a random curve described by the Schramm-Loewner evolution (SLE). This allows us to develop a theory of quantum fractal measures, consistent with the Knizhnik-Polyakov-Zamolochikov relation (KPZ), and to analyze their evolution under conformal welding maps related to SLE. As an application, one can construct quantum length and boundary intersection measures on the SLE curve itself. (Joint work with Scott Sheffield (MIT).)

Title: Quantitative Characteristics of Cantor Sets and Their Applications
Speaker: Anton Gorodetski, University of California-Irvine
Abstract:

In many cases the Cantor sets that appear in applications have zero Lebesgue measure. But there are other important quantitative characteristics of Cantor sets, such as Hausdorff dimension, box counting dimension, and thickness. We will discuss relations between these quantities, as well as some applications to conservative dynamics, celestial mechanics, and spectral theory of quasicrystals.
Title: Anomalous Diffusion in Living Cells  
Speaker: Denis Grebenkov, Ecole Polytechnique, France  
Abstract

The transport of macromolecules, vesicles and organelles inside living cells is a complex phenomenon with different regimes (sub-diffusive, super-diffusive, intermittent, transitory, etc.). In the plenary talk, we review some mathematical, physical and biological aspects of this phenomenon, with a special attention to recent progress and open mathematical problems in this field. In particular, we briefly describe several theoretical models of anomalous diffusion, including continuous-time random walks (with waiting times resulting from “caging” of a macromolecule in the overcrowded intracellular environment), fractional Brownian motion or generalized Langevin dynamics (with friction memory kernel), and diffusion on a fractal support (which mimics the complicated intracellular or intra-nuclear structure). We also discuss single-particle tracking (SPT) with optical tweezers that allows one to record 10-100 seconds long individual trajectories of tracers at time step up to 1 µs and spatial resolution up to few nm. While a huge amount of experimental data is available nowadays, their reliable statistical interpretation remains a challenging problem. In fact, as SPT techniques acquire only a limited number of random trajectories (generated by an unknown stochastic process), ensemble averages (or expectations) have to be replaced by time averages along an individual random trajectory. These are still random variables from which the dynamical, functional and micro-rheological properties of living cells have to be inferred. One deals with a very general problem of how to characterize a stochastic process from its random realization(s)? We present recent advances in this field and formulate some open mathematical problems.

Title: How the Genome Folds  
Speaker: Miriam Huntley, Harvard University  
Abstract:

How does DNA fold up inside the nucleus? The answer to this question has far-reaching implications for cell biology, yet studying the structure of DNA in vivo is a difficult task. Hi-C is a new experimental method designed to interrogate the three-dimensional structure of the genome. Using proximity-based ligation in conjunction with high-throughput sequencing, Hi-C produces high resolution contact maps of whole genomes. Hi-C contact maps led to the identification of genome-wide compartmentalization of open and closed chromatin. In addition, the Hi-C experiments showed that the chromosomal conformation at the megabase scale is consistent with the fractal globule, a densely packed but largely unknotted polymer structure. In this talk I will discuss these two concepts in detail, and review the latest developments from recent Hi-C experiments as the study of genome folding continues to unfold.
Title: Behind the Hofstadter's Butterfly: Chaos, Order, and Continuity.  
Speaker: Svetlana Jitomirskaya, University of California-Irvine  
Abstract:  
Up until the mid-70s the kind of spectra most people had in mind in the context of theory of Schrodinger operators were spectra occurring for periodic potentials and for atomic and molecular Hamiltonians. Then evidence started to build up that "exotic" spectral phenomena such as singular continuous, Cantor, and dense point spectrum do occur in mathematical models that are of substantial interest to theoretical physics. One area where such exotic phenomena are particularly abundant is quasi-periodic operators. They feature a competition between randomness (ergodicity) and order (periodicity), which is often resolved at a deep arithmetic level. Mathematically, the methods involved include a mixture of ergodic theory, dynamical systems, probability, functional and harmonic analysis. The interest in those models was enhanced by strong connections with some major discoveries in physics, such as integer quantum Hall effect, experimental quasicrystals, and quantum chaos theory, in all of which quasi-periodic operators provide central or important models. We will give a general overview concentrating on aspects where the competition and/or collaboration between order and chaos plays an important role, while highlighting recently developed dynamical methods.

Title: Analysis on Fractals  
Speaker: Jun Kigami, Kyoto University, Japan  
Abstract:  
The area "Analysis on Fractals" studies "Laplacians" ('differential'' operators in general) and diffusion processes on fractal objects. In mathematics such a rigorous treatment started about 25 years ago by the independent works of Kuauoka and Goldstein on constructions of "Brownian motions" on the Sierpinski gasket. Since then, there have been studies Constructions of Laplacian and/or diffusion processes, Eigenvales and eigenfunctions of Laplacians, Asymptotic behavior of associated heat kernels, uniqueness of geometrically symmetric diffusion processes and so on. In this talk, I will review some of those results mainly in the case of the Sierpinski gasket and mention applications and directions in the future.

Title: Fractional Gaussian Noise, Subdiffusion and Stochastic Networks in Biophysics  
Speaker: Samuel Kou, Harvard University  
Abstract:  
In recent years, single-molecule experiments have emerged as a powerful means to study biophysical/biochemical processes; many new insights are obtained from this single-molecule perspective. One phenomenon recently observed in single-molecule biophysics experiments is subdiffusion, which largely departs from the classical Brownian diffusion theory. In this talk, by introducing fractional Gaussian noise (i.e., the derivative of fractional Brownian motion) into the generalized Langevin equation, we propose a model
to describe the subdiffusion. We will study the analytical properties of the model, compare the model predictions with experiments, look at its connection with stochastic networks, and explore the implications of the model on enzyme reactions.

Title: Cuntz-Krieger Algebras and Wavelets on Fractals  
Speaker: Matilde Marcolli, California Institute of Technology  
Abstract:

We consider representations of Cuntz-Krieger algebras on the Hilbert space of square integrable functions on the limit set, identified with a Cantor set in the unit interval. We use these representations and the associated Perron-Frobenius and Ruelle operators to construct families of wavelets on these Cantor sets. This talk is based on joint work with Anna Maria Paolucci.

Title: Self-Organization of Rocky Coast Scaling Morphology, Fractal or Not, and Statistics of Erosion Events.  
Speaker: Bernard Sapoval, Ecole Polytechnique, France  
Abstract:

We recently proposed a minimal dynamical model of rocky coast erosion which is able to reproduce the diversity of rocky coast morphologies and their dynamics. It follows from the idea that erosion can spontaneously create irregular seashores, but, in turn, the geometrical irregularity of the coast participates to the damping of sea-waves, decreasing the average wave amplitude and erosive power. The resulting mutual self-stabilization dynamics of the sea erosion power and coastal irregular morphology leads spontaneously the system to a critical dynamics. Our results indicate that rocky coast erosion and the statistical theory of percolation are closely related. In this framework, the sometimes fractal geometry of coastlines can be recovered and understood in terms of fractal dimension of the external perimeter of a percolation cluster.

From a more practical point of view, the analogy with percolation interfaces means that the coast constitutes a strong, but possibly fragile, barrier to sea erosion, emerging from a self-organized selection process. In this frame, the dynamics of rocky coasts is an erratic phenomenon featuring numerous small erosion events, but sometimes large dramatic collapses. Recent studies, based on historical as well as recent data, have indicated that the frequency of magnitude of erosion events display long tail distribution. In other words the time evolution of a coast morphology does not enter the classical category of Gaussian process, but rather that of critical systems in physics.
Title: Coopetitive Dynamics in Evolutionary Game Theory  
Speaker: David Carfi, University of Messina, Italy  
Abstract:

In this talk, we shall deal with the dynamical aspects induced by coopetition in Evolutionary Game Theory and, particularly, with the dynamics in the coopetitive extensions of a normal-form game, in view of their evolutionary possible issues. Several dynamical processes and set-valued dynamical processes are generated by a single coopetitive game. We shall concentrate our attention, specifically, on the Nash coopetitive paths, on the payoff coopetitive multi-path, on the Pareto coopetitive multi-path and on the evolutionary stable strategy coopetitive paths of a coopetitive game, in consideration of their possible applications to Economics and Biology.

Title: Hyperfunctions and Spectral Zeta Functions of Laplacians on Fractals.  
Speaker: Nishu Lal, University of California-Riverside  
Abstract:

We will discuss the spectral zeta functions of fractal differential operators, in particular the Sturm-Liouville operator associated with a self-similar measure on the half-line. We obtain a factorization of the spectral zeta function expressed in terms of the Dirac delta hyperfunction and the zeta function associated with the dynamics of the corresponding renormalization map induced by the decimation method, viewed as a rational function on the complex projective plane. Furthermore, we conclude by showing that a similar factorization formula exists for the infinite Sierpinski gasket.

Title: Impurity Induced Non-Abelian Gauge Fields, Self-Similarity and Low Frequency Modes in Two Dimensional Atomic Molecular Systems  
Speaker: Michael Maroun, University of California-Riverside  
Abstract:

In the past few years it has become apparent that unique two dimensional atomic-molecular systems such as graphene lend themselves to novel and interesting prospects for modern nano-technology. In particular, it has been noted by Vozmediano, Katsnelson and Guinea that application to gauge field theory for such systems with specific types and configurations of impurities lead to non-trivial two dimensional quantum field theoretic models with interesting topological properties. The impurities can be regarded as microscopic corrugations as noted by Katsnelson and Geim independently. I investigate the basic two dimensional boundary value problem for nonlinear scalar fields analogous to non-Abelian gauge theories subject to fractal boundary conditions. The goal is to investigate the possibility that nano-devices, despite their size and owing to the onset of self-similarity, give rise to the existence of low frequency modes typically only present in large cavities. It is suggested that the onset of self-similarity creates “phantom” copies of
the nano-cavity allowing for standing waves of frequencies lower than expected for the scale of the physical device. This has wide spread applications, including but not limited, to nano-inductors for low frequency systems such as audio devices.

Title: Statistical Properties of Gibbs-Equilibrium Measures  
Speaker: Stephen Muir, University of California-Riverside  
Abstract:

This will be an expository talk intended to introduce the concept of a Gibbs probability measure as it occurs in two settings: classical lattice models of statistical physics and distance expanding dynamical systems. General existence and characterization theorems will be provided, and then some of the statistical properties that distinguish this class of probability measures will be described. In the lattice setting the focus will be on large deviations estimates and in the dynamical setting the focus will be on the central limit theorem and related asymptotic properties.

Title: Sequences of Compatible Periodic Hybrid Orbits of Prefractal Koch Snowflake Billiards.  
Speaker: Robert Niemeyer, University of California-Riverside  
Abstract:

The billiard table with a nowhere differentiable boundary is not well defined; the law of reflection holds at no point of the boundary. Denoting the Koch snowflake by KS, the billiard Omega(KS) is a canonical example of such a table and the focus of the talk. We will show that KS being approximated by a sequence of rational polygons and Omega(KS) being tiled by equilateral triangles both allow us to construct what we call a sequence of compatible periodic hybrid orbits. Under certain situations, such sequences have interesting limiting behavior indicative of the existence of a well-defined billiard orbit of Omega(KS). In addition to this, we provide a topological dichotomy for a sequence of compatible orbits. Other important properties and interesting results will be discussed, especially with regards to the possible presence of similarity in what we believe to be a well-defined periodic hybrid of the Koch snowflake fractal billiard Omega(KS). Finally, we will briefly discuss our results in the context of other fractal billiard tables and future research problems.

Title: Diffusion Metrics on Networks and Applications in Mathematical Statistics  
Speaker: Erin Pearse, Oklahoma State University  
Abstract:

Diffusion metrics can be used to reveal the intrinsic geometry of high-dimensional datasets by comparing the dispersion rate of a random walk from different starting locations. This is expressed precisely in terms of the heat kernel (or a similar kernel). Diffusion metrics initially arose in manifold learning as a framework for dimensionality reduction, clustering (or coarse-graining), and dataset parametrization. I will give an introduction to the basic machinery and give some examples demonstrating how diffusion metrics can be used to
obtain a dimensionality reduction via an eigenmapping technique, and also how one might obtain a very low-dimensional parametrization of a certain subset of \( \mathbb{R}^{10,304} \). I will also describe a current project (joint with some graduate students from quantitative psychology) which applies these methods to analyzing retention/dropout patterns in a university student population. Theoretical (mathematical) applications will also be apparent.

**Title:** Self-Similar Multifractal Measures and Their Zeta Functions  
**Speaker:** John Rock, California State Polytechnic University-Pomona  
**Abstract:**

In this talk, we discuss the construction and an analysis of a class of multifractal measures called self-similar measures. These measures play important roles in the mathematical modeling of highly irregular natural phenomena such as aggregates, snowflakes, turbulence, and fractal viscous fingering, to name a few. Our approach to the multifractal analysis of such measures is motivated on one hand by a classical approach known as symbolic dynamics and, on the other hand, by the theory of complex dimensions of fractal strings. In particular, through the notion of scaling regularity we define a family of fractal strings whose scaling zeta functions allow us to determine a type of multifractal spectrum for a given self-similar measure.

**Title:** Persistence and Domain Size for a Single Population in River Systems  
**Speaker:** Jonathan Sarhad, University of California-Riverside  
**Abstract:**

Linear differential equations in a one dimensional domain are often used to study the relationship between persistence and domain size in river system populations. However, scaling an interval can ignore that, at larger scales, a river system's geometry resembles a tree rather than an interval and that habitable cross sectional areas may vary throughout the system. Our question is how and when scaling up a tree structure affects persistence differently than scaling up an interval. We use a diffusion-advection equation in a metric tree graph, with linearized growth rate around the zero steady state, to identify when a single population will grow at low density, by doing a principal eigenvalue analysis in terms of domain scaling parameters and advection speeds. The metric graph represents a continuous tree system where edges represent actual domain; this mathematical framework is commonly referred to as a quantum graph. We primarily assume a lethal downstream boundary condition and interior junction and upstream boundary conditions which do not allow organisms in or out of the domain. We scale the tree domain by adding branching levels, with various assumptions on cross section and branch length decay going upstream in the system. Our results indicate that only under a special cross section assumption does the tree system scale the same as the one dimensional domain. In other cases, scaling the one dimensional domain can either underestimate or overestimate persistence depending on how the one dimensional domain is considered to scale. In a specific example, assuming diffusion only, a constant cross section, bounded edge lengths, and lethal
conditions at all boundary points, the principal eigenvalue is bounded below uniformly over increases in tree size. This is in contrast to the corresponding one dimensional problem whose principal eigenvalue can be driven to zero with increasing interval size. In addition, using numerical methods we quantify the decay of the principal eigenvalue with increases in domain size in the different systems, comparing over various parameter assumptions.

Title: Casimir Effect on Laakso Spaces  
Speaker: Ben Steinhurst, Cornell University  
Abstract:

We define three dimensional Laakso spaces as the uncountably many unit cubes that have been identified so as to be able to cross from one cube to another, from one ‘universe’ to another. On these spaces we construct a Laplacian and derive a spectral zeta function from which can be calculated the strength of a Casimir effect (work done with Robert Kesler).

Title: Uniform Fractal Properties of Some Quasiperiodic Models  
Speaker: William Yessen, University of California-Irvine  
Abstract:

Fractal structures in quasiperiodic models (such as fractal spectrum of quasiperiodic Jacobi matrices, or fractal energy spectrum for Ising quasicrystals) have become somewhat of an expected phenomenon over the course of the past couple of decades of active investigation of these models. Fractality, so to speak, is a purely topological description, and while difficult to prove, once established it requires finer analysis in connection with relevant properties of the aforementioned models; for example, properties of fractal dimensions (such as regularity depending on the parameters of a given system, lower and upper estimates, equality or lack thereof of different fractal dimensions---such as Hausdorff vs. box-counting---and so on). On the other hand, as ongoing research has demonstrated, there are often intimate connections with the theory of dynamical systems. A prototypical example of a quasiperiodic model where smooth dynamical systems appear naturally and carry nontrivial implications are the so-called Fibonacci models, such as the Schroedinger operator on the one-dimensional integer lattice with quasiperiodic potential generated by the so-called Fibonacci substitution on two letters; or orthogonal polynomials on the unit circle with Fibonacci Verblunsky coefficients. In this talk, we'll describe a few such models (in addition to the two mentioned above, we'll discuss Fibonacci Ising models and Fibonacci Jacobi matrices), and we'll describe the techniques from dynamical systems that have been developed over the past almost three decades (and some quite recently: in the past two years) that can be used to aid with detailed analysis of relevant fractal structures appearing in these models.