1. (Chapter 1 supplementary, problem 7): There are 12 men at a dance. (a) In how many ways can eight of them be selected to form a cleanup crew? (b) How many ways are there to pair off eight women at the dance with eight of those 12 men?

2. (Chapter 1 supplementary, problem 10): Mr. and Mrs. Richardson want to name their new daughter so that her initials (First, middle, and last) will be in alphabetical order with no repeated initial. How many triples of initials can occur under these circumstances?

3. (Chapter 1 supplementary, problem 12): In how many ways can a teacher distribute 12 different science books among 16 students if (a) no student gets more than one book? (b) the oldest student gets two books but no other student gets more than one book?

4. (Chapter 1 supplementary, problem 16a): Find the coefficient of $x^2yz^2$ in the expansion of $[(x/2) + y - 3z]^5$

5. (Section 4.1, problem 15): Prove that for every positive integer $n > 4$, we have that $n^2 < 2^n$.

6. Find a pair of integers $(x, y)$ for which $19x + 47y = 200$.

7. (Section 5.5, problem 4): Let $S = \{3, 7, 11, 15, 19, \ldots, 95, 99, 103\}$. How many elements must we select from $S$ to insure that there will be at least two whose sum is 110?

8. (based on Section 5.5, problem 13): Alice and Bob are playing a game. Alice gives Bob a list of five positive integers, each of which is at most 9. Bob wins if he can find two different subsets of that list having the same sum. For example, if Alice chooses 1, 2, 4, 5, 7, Bob can choose \{1, 2, 4\} and \{7\}, both of which add to 7. Show that no matter what set of 5 integers Alice chooses, Bob can still win.

9. Find the number of ways of arranging the letters in $MATHEMATICS$ so that no pair of consecutive letters appears.

10. Find the number of ways of arranging the letters in $MATHEMATICS$ so that exactly two pairs of consecutive letters occurs (this uses Theorem 8.2 – if necessary that theorem will be on your exam).

11. Determine the generating function for the sequence 1, 2, 2, 8, 16, 32, 64, where the formula $a_n = 2^n$ holds for every term except the third one.

12. Find a generating function for the number of ways of distributing $n$ pennies among five children such that every child gets at least 2 cents and the youngest child gets at most 5 cents.

13. Find the $x^5$ coefficient of $\frac{x}{1-3x}$.
14. Find the constant (the \(x^0\) coefficient) of \((3x^2 - x^{-1})^{15}\).

15. Write down, but do not solve, a recurrence relation for the number of ways of writing \(n\) as a sum of 2's, 3's, and 5's. Assume that order matters, that is that 2 + 3 and 3 + 2 are different.

16. Solve: \(a_n - 3a_{n-1} + 2a_{n-2} = 2^n, a_0 = 0, a_1 = 1\).

17. Draw a 3-regular graph on 25 vertices, or explain why no such graph exists.

18. Determine whether the graphs in figure 11.42 in page 537 of your book are isomorphic or not.

19. For which \(n\) does the Hypercube \(Q_n\) have an Eulerian trail?

20. Is graph (f) on page 554 of your book planar? Why or why not?

21. (11.4, problem 19): Let \(G\) be a loop-free, connected, 4-regular, planar graph with 16 edges. How many regions are in a planar drawing of \(G\)?

22. 16 politicians get together at a party. It is known that each politician despises exactly 7 of his colleagues at the party. Show that we can put all 16 politicians in a receiving line so that nobody has to stand next to someone he or she despises.

23. A class of 5 people are trying to divide up into teams to work on at most 3 different projects. It is known that (Alice and Bob), (Bob and Charlie), (Charlie and Dave), (Dave and Alice), (Alice and Edna), and (Edna and Bob) are all incompatible pairs...putting both people in a pair on the same project would be a disaster. a) Express this in terms of a graph-coloring problem. b) Use chromatic polynomials to find the number of different arrangements of teams possible.

24. Prove that any graph on 10 vertices with at least 26 edges must have chromatic number at least 3.

25. Find the tree on 8 vertices with Prufer code 161833

26. Problem 3 from section 13.1

27. (based on 13.4, number 2): Cathy is liked by Albert, Joseph, and Robert; Janice by Joseph and Dennis; Theresa by Albert and Joseph; Nettie by Dennis, Joseph, and Frank; and Karen by Albert, Joseph, and Robert. USING THE ALGORITHM FROM THE PROOF OF HALL’S THEOREM, find a way of setting up all five women up on dates with men who like them. Be sure to show and explain your procedure.
28. (13.4, number 7): Fritz is in charge of assigning students to part-time jobs at the college where he works. He has 25 student applications, and there are 25 different part-time jobs available on the campus. Each applicant is qualified exactly four jobs, but each job can be performed by at exactly four applicants. Can Fritz assign all the students to jobs for which they are qualified? Explain.