Assignment 11 due 11/5

1. For Section 10.4, problem 1cd, find the generating functions for the sequences generated by the given recurrence. You do NOT need to find the sequences themselves!

2. Let $P_n$ be the regular polygon with $n$ vertices (so $P_4$ is the square, $P_5$ is the regular pentagon, etc.). A triangulation of $P_n$ is a division of $P_n$ into $n-2$ triangles using the diagonals of $P_n$. For example, the two triangulations of $P_4$ are shown below.

Note that these are considered two different triangulations even though you can get one just by rotating the other.

a) Draw all the triangulations of $P_5$.

b) Let $a_n$ be the number of triangulations of $P_{n+2}$, and define $a_0 = 1$. Show that $a_n$ satisfies the recurrence

$$a_n = a_{n-1}a_0 + a_{n-2}a_1 + a_{n-3}a_2 + \ldots a_1a_{n-2} + a_0a_{n-1}.$$ 

This is the same recurrence that we showed in class was satisfied by the number of ways of drawing $n$ semicircles, a sequence we know is counted by the Catalan numbers. Together with the initial conditions, this shows that the number of triangulations of an $n+2$-gon is given by the Catalan numbers.

(Hint: Every triangulation has to have exactly one triangle containing the edge between $n+2$ and 1. Divide the triangulations into classes depending on which triangle contains this edge).

3. Section 11.1, problems 2, 10

4. Section 11.2, problems 4, 9, 12