

MATH 171
MIDTERM REVIEW

You should use this outline to help you organize your studying for the upcoming midterm exam. Ideally you should understand everything listed here. Of course, not all of this material will necessarily be on the exam and there could be material on the exam that is not mentioned here.

This is essentially a summary of what we have covered so far. Roughly, it falls into 3 categories: basic theory, computation, classification. It is very important to see how these three things are interconnected.

1 Basic theory

1. Set cardinality, bijective maps, Definition 0.13, the cardinality of \mathbb{Z} and \mathbb{Q} versus, say \mathbb{R} , \mathbb{R}^+ , \mathbb{R}^* , and \mathbb{C} .
2. The definition of equivalence relations, partitions, and the relationship between the two. Know what equivalence classes are for equivalence relations defined on groups, what they “look” like (e.g. Quiz 2). Know and understand the proof of Theorem 0.22
3. Know the definition of a binary operation on a set, know how to show binary operations are well-defined, be able to prove commutativity and associativity, be comfortable with questions like exercises 26-37 in Section 2.
4. Know how to build isomorphisms: examples 3.8, 3.9, 3.10, and how to show two binary structures, groups, etc. are not isomorphic, exercises 11-18 in section 3.
5. You must know the basic properties of a group. What are the axioms? Are inverses unique? Is the identity unique? What is the order of a group element? (Theorems 4.15, 4.16, 4.17)
6. Know how to prove a subset of a group is a subgroup (exercises 1-13 section 5). What is a proper subgroup? Are there groups with no proper subgroups?
7. Cyclic groups: What’s a generator? What’s the minimal set of generators needed to generate a cyclic group? Is that set of generators necessarily unique?

8. Cyclic groups: Theorem 6.1, 6.6, (understand the proofs) Theorem 6.14 (very important!) Corollary 6.16.
9. Permutation groups: What's the permutation group of a set? Why is it a group? (Theorem 8.5). What's the definition of S_n ? How do you visualize its elements? What's D_n ? How is it related to S_n ?
10. Cayley's theorem: How does it work, what does it say? Know the proofs of Lemma 8.15, Theorem 8.16.

2 Computation tools

1. Modular arithmetic, operations of matrices (transpose, inverse, determinant, exercises 8-13 in section 5)
2. Basic operations in groups (left-right cancellation).
3. Playing with elements of cyclic groups (elements that look like na or a^n .)
4. Basic facts about complex numbers, roots of unity. (I was going to prepare a separate handout on this, but I noticed your book already has a good review of this stuff: see pages 12-18.)
5. Computing the gcd of small numbers, computing the order of a group element.
6. Subgroups: Compute all the subgroups of various cyclic groups and their orders. Exercises 22-29 section 6
7. Techniques you can use when you're dealing with finite groups and subgroups: exercises 29, 34 in section 5, 48 in section 6, 49 in section 8.
8. Computation in S_n : Composing elements, finding inverses, computing the order of elements, constructing cyclic subgroups. (For example, exercises 1-20 in section 8.

3 Classification

1. Understand all the tools needed to solve Problem 10 in section 8.

2. Exercises 33-37 in section 6, the variants I discussed in class and office hours, Quiz 3.
3. Classification and structure of cyclic groups. Understand why: Every infinite cyclic group is “just” \mathbb{Z} , any finite cyclic group is “just” \mathbb{Z}_n . (What does “just” really mean here?) How does the order of a subgroup of a cyclic group relate to the order of the bigger group? What’s special about \mathbb{Z}_p where p is prime?
4. Know the Klein 4 group inside and out. Why is it interesting? Why isn’t it cyclic? What do its subgroups look like?
5. Can you classify all groups with, say, order ≤ 6 ? Are they all abelian, all cyclic, etc.?
6. Know examples of finite and infinite non-abelian groups. Be able to demonstrate that they are not abelian.
7. Know some abelian groups that aren’t cyclic. Why aren’t they cyclic?
8. What kind of subgroups does S_3 have? Can you build the subgroup lattice? Can you describe a few interesting subgroups of S_4 ?
9. What’s the relationship between the number of generators of a cyclic group and the order of a cyclic group?