Quiz #2 Solutions

#1] See discussion lecture notes (online)

#2] Let (m,n) & $(n,\ell) \in R$. Then $m = 5k_1 + r$
$n = 5k_2 + r$

& $n = 5k_1' + r'$
$\ell = 5k_2' + r'$ For some k_1, k_2, k_1' & $k_2' \in \mathbb{N}$.

By uniqueness of division algorithm, $n = 5k_1' + r' = 5k_2 + r$
$\Rightarrow r' = r$. (<u>or</u>: $n = 5k_1' + r' = 5k_2 + r \Rightarrow 5(k_1' - k_2) = (r - r')$
$\Rightarrow 5$ divides $(r - r')$. But r & $r' \leq 4 \Rightarrow r - r' = 0 \therefore r = r'$)

$r' = r \Rightarrow (m,\ell) \in R$ by definition of R.

#3] Let $\{[x]\}$ be the family of equiv. classes. need to show:
(1) $X = \bigcup_{x \in X} [x]$ & (2) if $[x] \cap [y] \neq \phi$ then $[x] = [y]$

(1) Let $z \in X$. Claim $z \in [z]$: by def. $[z] = \{y \in X \mid (y,z) \in E\}$.
E is equiv. relation $\therefore$ E is symmetric. i.e. $(z,z) \in E$. $\therefore z \in [z]$.
So $z \in \bigcup_{x \in X} [x]$, which implies $X \subseteq \bigcup_{x \in X} [x]$. Each equiv. class $[x]$ is a subset of X $\therefore$ $X = \bigcup_{x \in X} [x]$

(2) Suppose $[x] \cap [y] \neq \phi$. $\therefore \exists z \in [x] \cap [y] \Rightarrow (x,z)$ & $(y,z) \in R$.
Since E is symmetric & transitive, $(x,y) \in R$ $\therefore [x] = [y]$ $\square$

②

#4] Assume $(a_1, a_2) \sim (b_1, b_2)$ & $(b_1, b_2) \sim (c_1, c_2)$.

$\therefore$ by def of $\sim$ $a_1 b_2 = a_2 b_1$ & $b_1 c_2 = b_2 c_1$

$\therefore$ $a_1 \cancel{b_2} \cancel{b_1} c_2 = a_2 \cancel{b_1} \cancel{b_2} c_1 \Rightarrow a_1 c_2 = a_2 c_1 \Rightarrow$

$(a_1, a_2) \sim (c_1, c_2)$ $\therefore$ $\sim$ transitive.