

**UNIQUE LIFT OF AN ACTION OF THE  
TEMPERLEY-LIEB ALGEBRA TO A FAITHFUL ACTION  
OF THE FUSS-CATALAN ALGEBRA**

ABSTRACT. In his thesis F. Hivert introduced a faithful action of the symmetric group  $S_n$  on the ring of polynomials  $\mathbb{C}[x_1, x_2, \dots, x_n]$ , which does not preserve the multiplication, but still leads to interesting results. The invariants of the action are the quasi-symmetric functions. Unfortunately, when we extend this action to the group algebra  $\mathbb{C}[S_n]$ , the action we obtain is not faithful anymore. However, by taking the quotient of  $\mathbb{C}[S_n]$  by the kernel of this new action we obtain a faithful action of the Temperley-Lieb  $TL_n(2)$  on the ring of polynomials in  $n$  variables. Since the Temperley-Lieb algebra can be embedded in the Fuss Catalan algebra on two colours  $FC_n(a, b)$  with  $ab = 2$ , a natural question to ask is if the action of the Temperley-Lieb algebra can be extended to a faithful action of the Fuss-Catalan algebra, and if the extension is unique. We will show how we use the theory of subfactors to answer both these questions in the affirmative. This is joint work with R. Burstein.