THE MATHEMATICS OF PLANET EARTH

John Baez
British Mathematical Colloquium
25 March 2013
Robert Fogel - *The Escape from Hunger and Premature Death, 1700-2100*
Atmospheric Carbon Dioxide
Measured at Manua Loa, Hawaii

The Keeling Experiment — Global Warming Art
Antarctic ice cores and other data — Global Warming Art
Reconstruction of temperature from 73 different records — Marcott et al.
The climate gamble:

This is based on a recent MIT paper comparing a world where we continue what we’re doing, and a world where we take aggressive action.
With just 3°C of warming, the US National Academy of Sciences expects that:

- 9 out of 10 northern hemisphere summers will be warmer than 1 out of 10 in 1980-2000.
- Much more land will be burned by wildfires in parts of Australia, Eurasia, and North America.
- Extreme precipitation events will increase by 9-30%.
- Rainfall in some dry regions will drop by 15-30%.

Furthermore, species are already moving 6 kilometers closer to the poles each decade, and the oceans are becoming more acidic. The rate of extinction, already about 10 times its average level, will increase.
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What can mathematicians do?

1. Teach math better.
2. Fly less. I burnt 0.6 tonnes of carbon by flying here. In 2011 the average person on Earth burnt 1.2 tonnes.

I usually give this talk virtually... and that's what I should have done.

One hard thing:

Invent the math we need for life on a finite-sized planet.
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But let’s go back and see how math played a role in an even bigger revolution: the Agricultural Revolution.

During this revolution, from 10,000 to 5,000 BC, we began to systematically exploit solar power by planting crops.

By now we use about 25% of all plant biomass grown worldwide! If this reaches 100% there will be, in some sense, no ‘nature’ separate from humanity.
Starting shortly after the end of the last ice age, the agricultural revolution led to:

- surplus grain production, and thus kingdoms and slavery.
- *astronomical mathematics* for social control and crop planning.
- *geometry* for measuring fields and storage containers.
- *written numbers* for commerce.

Consider the last...
Starting around 8,000 BC, in the Near East, people started using 'tokens' for contracts: little geometric clay figures that represented things like sheep, jars of oil, and amounts of grain.

The Schøyen Collection
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Eventually they gave up on the tokens. The marks on tablets then developed into the Babylonian number system! The transformation was complete by 3,000 BC.
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J. J. O’Connor and E. F. Robertson, Babylonian Numerals
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By 1700 BC the Babylonians could compute $\sqrt{2}$ to 6 decimals:

$$1 + \frac{24}{60} + \frac{51}{60^2} + \frac{10}{60^3} \approx 1.414213...$$

Yale Babylonian Collection, YBC7289
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Math may undergo a transformation just as big as it did in the Agricultural Revolution.
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So, this machine should be self-reproducing. It should turn some of the CO$_2$ into new machines.

Even better, these machines should spread without human intervention.
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This is a simple example of **ecotechnology**: technology that works like nature and works with nature.

For sophisticated ecotechnology we need to pay attention to what’s already known—*permaculture*, *systems ecology* and so on. But better mathematics could help.
To understand ecosystems, ultimately will be to understand networks. — B. C. Patten and M. Witkamp
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My own work on networks is motivated by ecology, but it’s rather abstract, so I won’t talk about it here.
Let’s look at something more concrete.
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Is there math in a leaf?
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Is there math in a leaf?

Yes! A mathematician at U.C. Davis, Qinglan Xia, has written a paper called *The Formation of a Tree Leaf*. 
He models a leaf as a union of square cells centered on a grid, together with ‘veins’ forming a weighted directed graph from the centers of the cells to the root. The leaf grows new cells at the boundary while minimizing a certain function.
The function depends on two parameters. Changing these gives different leaf shapes:
Qinglan Xia’s work is definitely math:

Lemma 3.8. Suppose \((\Omega, G)\) is an \((\epsilon, h)\) leaf and \((\mu, \Theta) = \phi_h (\Omega, G)\). Then the total mass of the Radon measure is bounded above by

\[
M(\mu) \leq \pi (R_\epsilon + h)^2
\]

and the total variation of the vector measure \(\Theta\) is bounded by

\[
M(\Theta) \leq \epsilon \pi^{2-\alpha} (R_\epsilon + h)^{4-2\alpha}.
\]

Proof. Since \(\Omega \subset B_{R_\epsilon} (0)\), the mass of \(\mu\) is given by

\[
M(\mu) = ||\Omega|| h^2 = \text{area}\left( \bigcup_{x \in \Omega} \left\{ x + \left[ \frac{-h}{2}, \frac{h}{2} \right] \times \left[ \frac{-h}{2}, \frac{h}{2} \right] \right\} \right)
\]

\[
\leq \text{area} (B_{R_\epsilon + h} (0)) = \pi (R_\epsilon + h)^2.
\]

Also, since \(w(e) \leq ||\Omega|| h^2\) for each \(e \in E(G)\), the total variation of \(\Theta\) is given by

\[
M(\Theta) = \sum_{e \in E(G)} w(e) \text{length} (e)
\]

\[
\leq (||\Omega|| h^2)^{1-\alpha} \sum m_\beta (e^+) (w(e))^\alpha \text{length} (e)
\]
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It’s just beginning to be born. I hope you can help out. Check out the Azimuth Project!