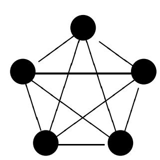
Loop Quantum Gravity, Quantum Geometry and Spin Foams

John C. Baez

GR17
Dublin
July 20, 2004

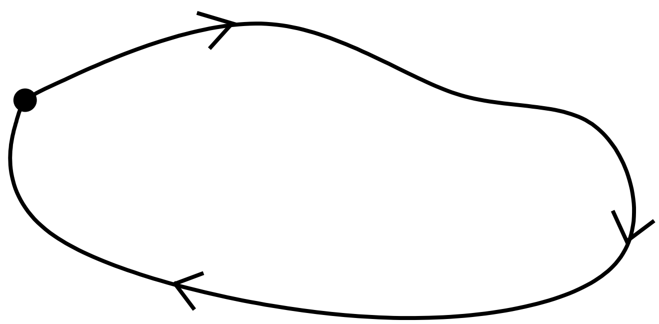


Loop Quantum Gravity

Loop quantum gravity tries to combine general relativity and quantum theory in a *background-free* theory. So, we cannot take gravitons, strings, etc. moving on a spacetime with a pre-established geometry as the basic building blocks of the theory. Instead, we must start with *quantum states of geometry.*

To describe these, we ask:

What is the amplitude for a spinning test particle to come back to the state it started in when we parallel transport it around a loop in space?

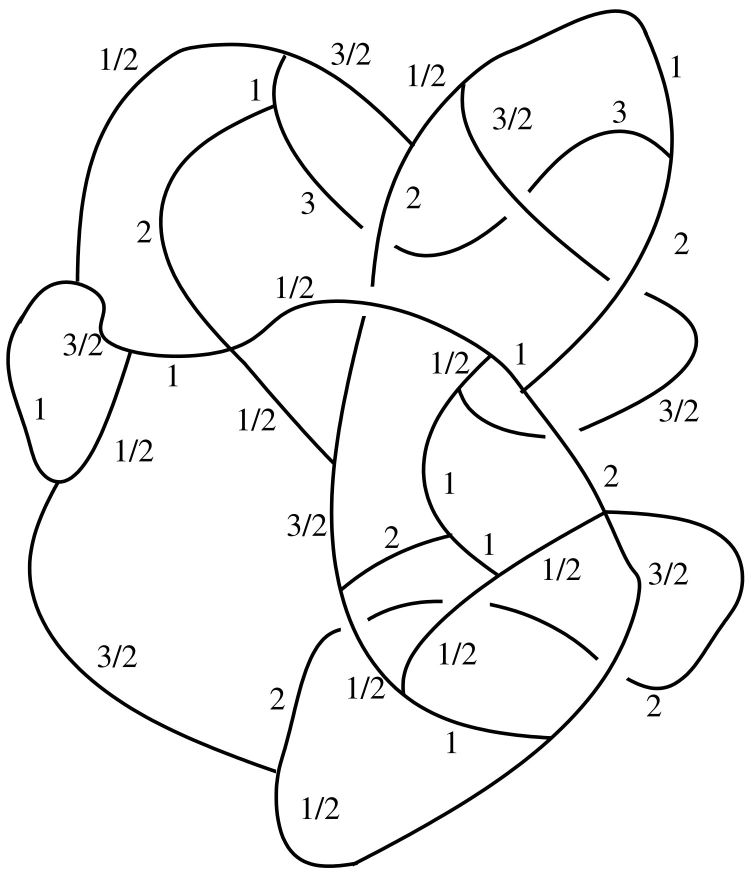


$j = 1/2, 1, 3/2, 2, \ldots$

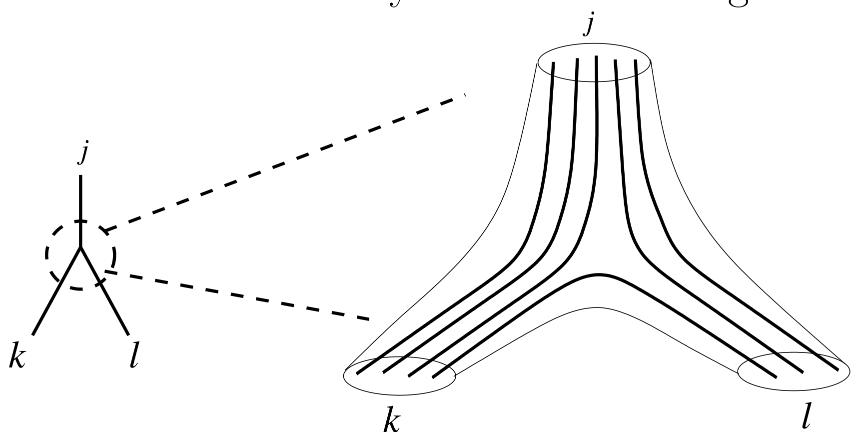
The answer doesn't depend on the starting point or the direction of the loop, so we can ignore those. It's enough to consider spin-1/2 particles, so *a state of quantum geometry assigns to each loop an amplitude — a complex number.*

Spin Networks

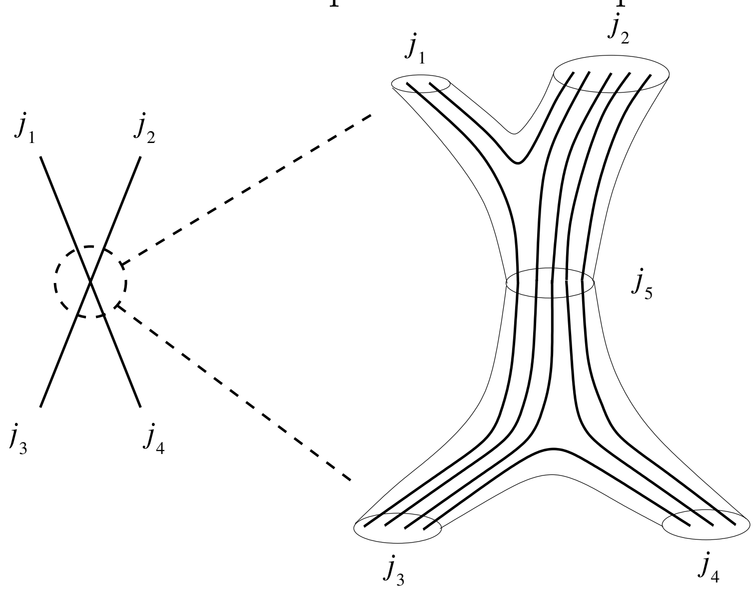
More generally, a state of quantum geometry assigns an amplitude to any system of spinning test particles tracing out paths in space, merging and splitting. These are described by *spin networks*: graphs with edges labelled by spins...



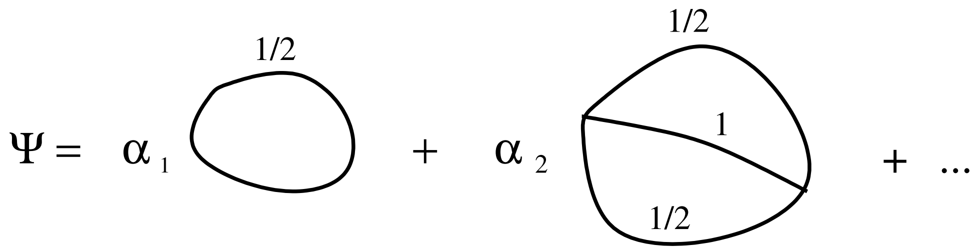
...together with 'intertwining operators' at vertices saying how the spins are routed. For vertices where 3 edges meet, there's at most one way to do this routing:



For vertices where more than 3 edges meet, we can formally 'split' them to reduce the problem to the previous case:



A quantum state of the geometry of space assigns an amplitude to any spin network. So, we can think of these states as *complex linear combinations of spin networks*, with these amplitudes as coefficients:



We could also use loops, but spin networks are an *orthonormal basis* of states, so they are more convenient.

What is the *meaning* of these spin network states? For this we must describe operators corresponding to interesting observables: lengths, areas, volumes...

We shall do this assuming that parallel transport is done using the real Ashtekar connection

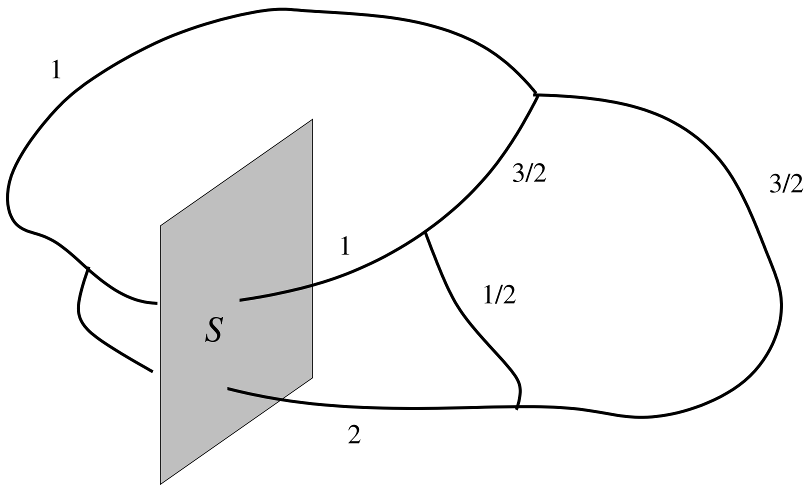
$$A = \Gamma - \gamma K$$

where Γ is the Levi-Civita connection on space, K is the extrinsic curvature and $\gamma \in \mathbb{R}$ is the Barbero–Immirzi parameter.

We shall only consider *area operators*....

Quantization of Area

If a spin network intersects a surface S transversely:



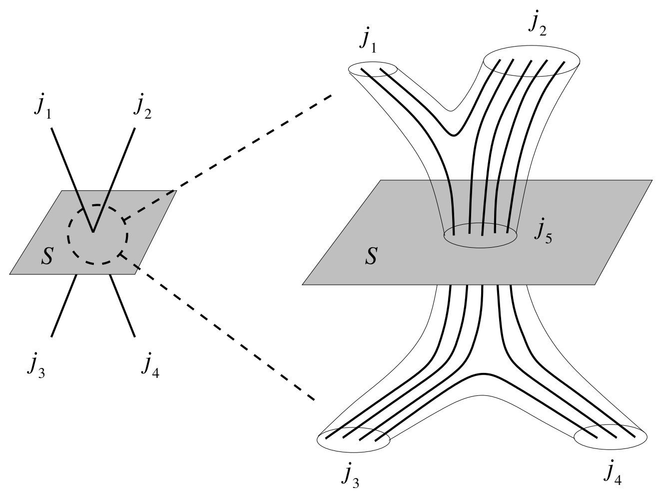
then this surface has a definite area in this state, given as a sum over the spins j of the edges poking through S:

$$\text{Area}(S) = 8\pi\gamma \sum_{\text{spins } j} \sqrt{j(j+1)}$$

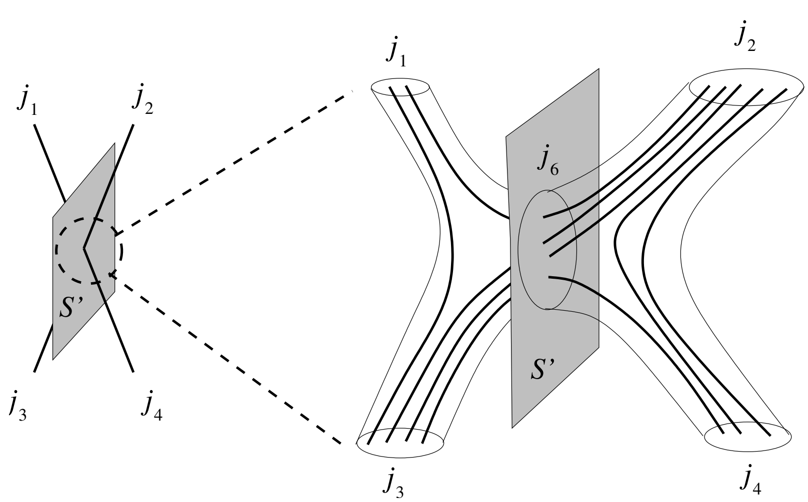
in units where the $\hbar = c = G = 1$. In particular, the operator for area has a *discrete spectrum*.

Uncertainty Principle for Area

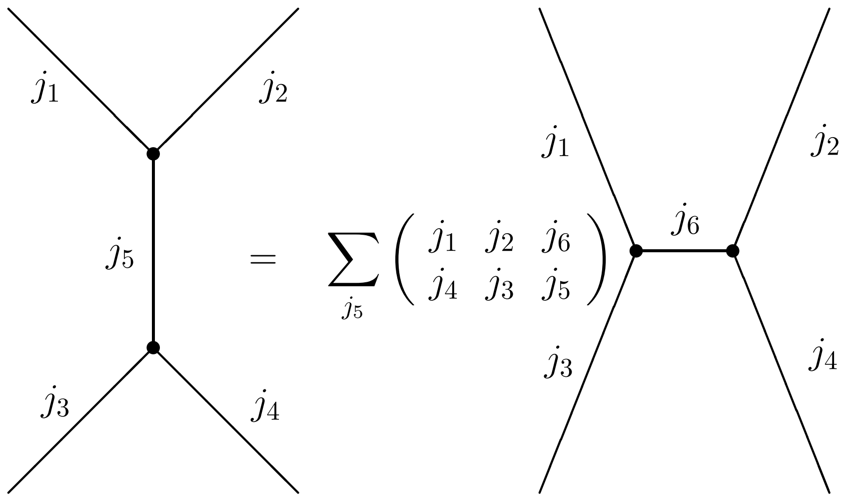
If a surface S intersects a spin network at a vertex, we must examine the routing to compute the area of S:



To describe states with definite areas, we must split the vertex so that the new edge intersects S transversely. This surface S' requires a different splitting:



Different splittings give different bases of states. To change from one basis to another we must use a matrix called the '$6j$ symbols':

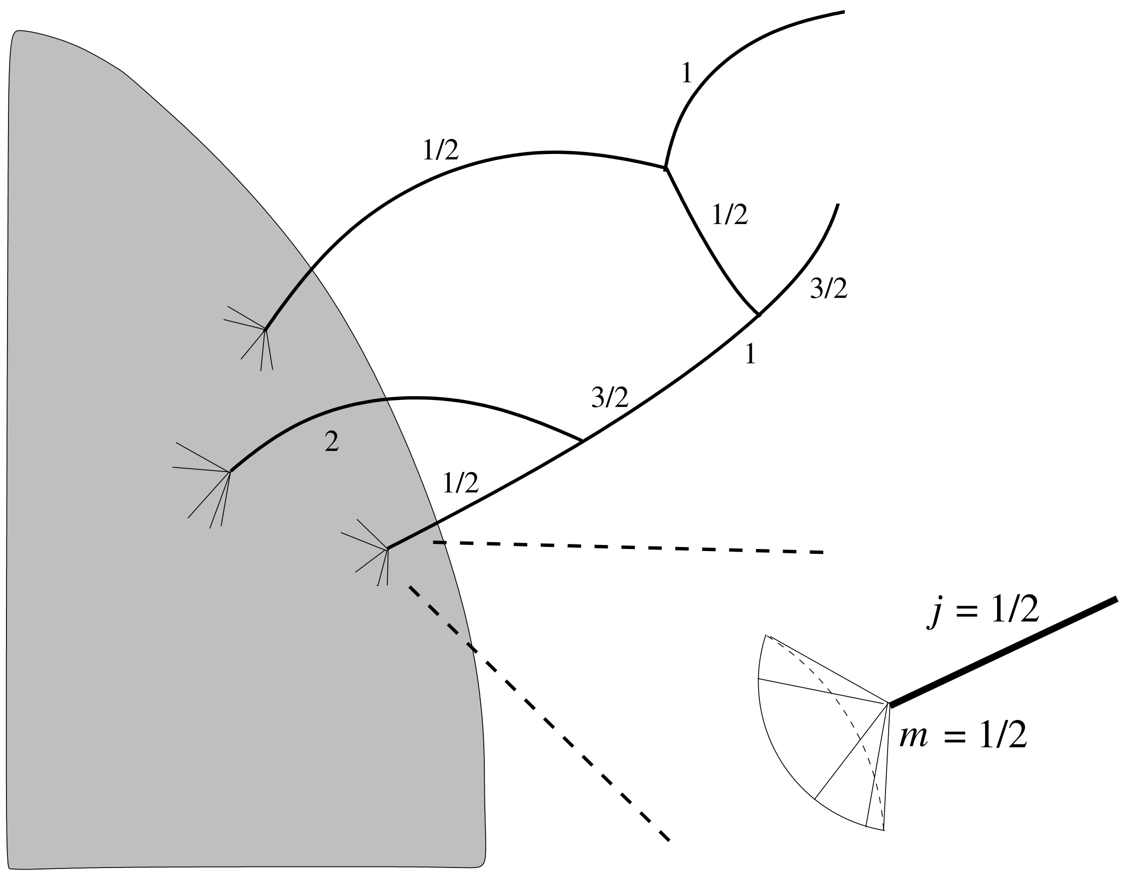


The area of S only has a definite value in the first basis of states, while that of S' only has a definite value in the second basis. *There is no basis of states in which the areas of both S and S' have definite values.*

In other words, the area operators for intersecting surfaces do not commute, so the uncertainty principle applies.

Black Hole Entropy

A careful analysis of 'isolated horizons' lets us do loop quantum gravity in the presence of a black hole. Spin network edges puncturing the horizon of a black contribute to its area. The intrinsic curvature of the horizon is concentrated at these punctures:



The curvature at a puncture is determined by a quantum number $m = -j, -j+1, \ldots, j$, where j is the spin of the edge piercing the horizon at this point. These quantum numbers specify a quantum state of a U(1) connection.

A quantum state of a U(1) connection on the horizon is determined by a list of numbers $m_i = \pm 1/2, \pm 1, \ldots$. If the black hole has area close to A, we must have

$$A \cong 8\pi\gamma \sum_i \sqrt{j_i(j_i+1)}$$

and thus

$$8\pi\gamma \sum_i \sqrt{m_i(m_i+1)} \lesssim A$$

since m_i ranges from $-j_i$ to j_i.

If we count the total number N of such states, for large A we find

$$N \sim e^{(\gamma_0/\gamma)\frac{A}{4}}$$

where γ_0 is a constant whose correct value has been determined only recently, by Domagala, Lewandowski and Meissner:

$$\gamma_0 = 0.2375329...$$

which is the real solution of

$$\sum_{m=\frac{1}{2},1,\frac{3}{2},\ldots} 2e^{-2\pi\gamma_0\sqrt{m(m+1)}} = 1$$

The probability that a puncture is labelled by $\pm m$ is

$$2e^{-2\pi\gamma_0\sqrt{m(m+1)}}$$

The black hole entropy is therefore

$$S = \ln N \sim (\gamma_0/\gamma)\,\frac{A}{4}$$

which matches Hawking's semiclassical calculation

$$S = \frac{A}{4}$$

if and only if the Barbero–Immirzi parameter is given by

$$\gamma = \gamma_0.$$

Thus, *agreement with semiclassical results forces a specific value for the 'quantum of area'*: with this γ, the smallest allowed area is

$$8\pi\gamma_0\sqrt{\tfrac{1}{2}(\tfrac{1}{2}+1)} = 5.17004...$$

The same calculation works for:

- charged, rotating and/or distorted black holes;
- black holes coupled to a dilaton field.

It also reproduces the usual semiclassical results for non-minimally coupled matter, where S is *not* just $A/4$ — still with the same value of γ!

Loop Quantum Cosmology

To understand dynamics in loop quantum gravity, we need states satisfying the Hamiltonian constraint:

$$H\Psi = 0$$

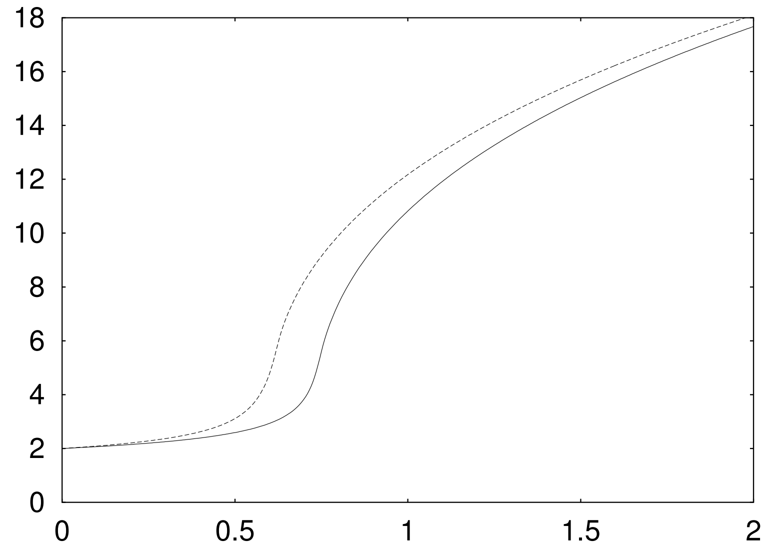
and we must *extract physics from them* — the hard part.

Thiemann has proposed a candidate for the operator H, and found solutions to $H\Psi = 0$, but to extract physics from them we need *semiclassical states*: states that closely approximate solutions of the classical Einstein equations.

Much work is being done on this. So far the most rapid progress has come by focusing on *highly symmetrical states*: homogeneous isotropic cosmologies. Bojowald has shown that by adapting Thiemann's work to this context:

- The state of the universe is a wavefunction Ψ depending on the 'size of the universe' $\mu \in \mathbb{R}$ and the values of various matter fields.
- The equation $H\Psi = 0$ is a *difference equation* in μ.
- This equation has well–behaved solutions which reduce to those of ordinary Wheeler-DeWitt quantum cosmology at scales $\mu \gg \ell_p$.
- We can extend these solutions to $\mu < 0$ — 'before the big bang'.

- Curvatures remain bounded, so there is no 'singularity'.
- There is typically an inflationary epoch near the Planck time, with a graceful transition to a Friedmann-like cosmology:



Here we see the radius $a(t)$ as a function of t for the effective Friedmann equation with a scalar field having $V(\phi) = 0$ (solid curve) or a quartic potential (dashed curve).

- In Bianchi IV models, chaotic behavior is suppressed by quantum effects near the Planck time.

Can we use cosmology to test loop quantum gravity?

Lorentz Violation?

Leaving the world of highly symmetric solutions, can we find semiclassical states that closely approximate Minkowski spacetime, while allowing quantum fluctuations of a general kind? A full answer will require a better understanding of *dynamics*, but various researchers have suggested that these semiclassical states might lack perfect Lorentz-invariance, leading to deviations in the energy-momentum relations:

$$E^2 = m^2 + |p|^2 + \kappa_1 \ell_p |p|^3 + \kappa_2 \ell_p |p|^4 + \cdots$$

Jacobson/Liberati/Mattingly, Konopka/Major and others have examined possible experimental tests, including:

- The (*missing?*) GZK cutoff on high-energy cosmic rays.
- The (*missing?*) cutoff on high-energy photons.
- Dispersion or vacuum birefringence of high-frequency radiation from distant sources.
- The vacuum Čerenkov effect, photon decay, and other Lorentz-forbidden processes.

These already highly constrain κ_1 for some species of particles, and κ_2 is within sight...

Can we use these ideas to test loop quantum gravity?

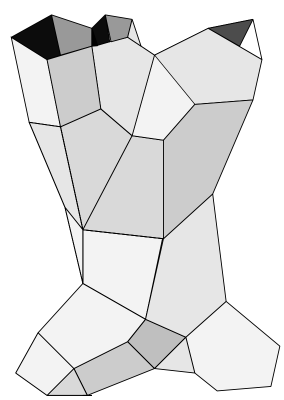
Taming Lorentz Violation?

Doubly special relativity, the κ-Poincaré group and other formalisms allow one to *deform* rather than merely *discard* Lorentz symmetry. The deformation parameter is a fundamental energy scale, say the Planck energy E_p, which is preserved by the analogue of Lorentz transformations.

Can we derive any of these as an effective limit of loop quantum gravity?

Quantum Spacetime via Spin Foams?

The idea: spacetime and everything in it is a quantum superposition of 'spin foams'. A spin foam is a generalized Feynman diagram where instead of a graph we use a 2-dimensional complex whose slices are spin networks:



A spin foam model specifies how to calculate an amplitude for any such spin foam — typically as a product of vertex amplitudes, edge amplitudes, face amplitudes, etc. There is a lot of beautiful mathematics here, but:

Can we use these to get a **spacetime picture** *of loop quantum gravity?*

Can we find a spin foam model whose behavior at length scales large compared to the Planck scale reduces to general relativity?