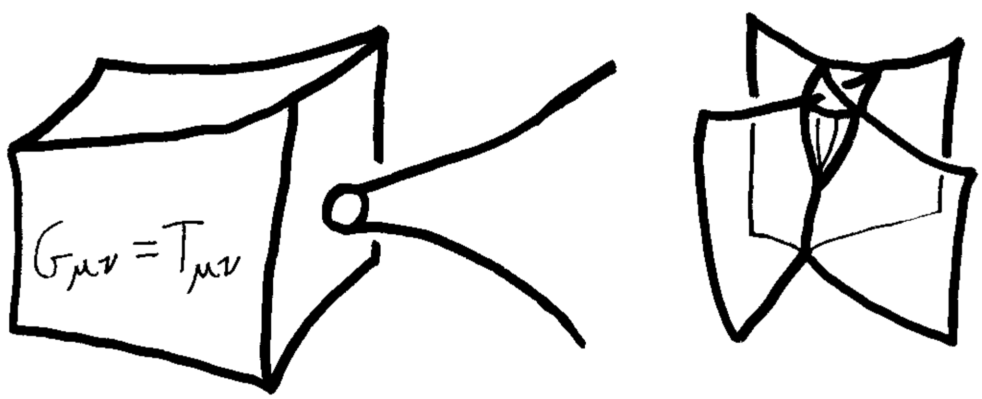
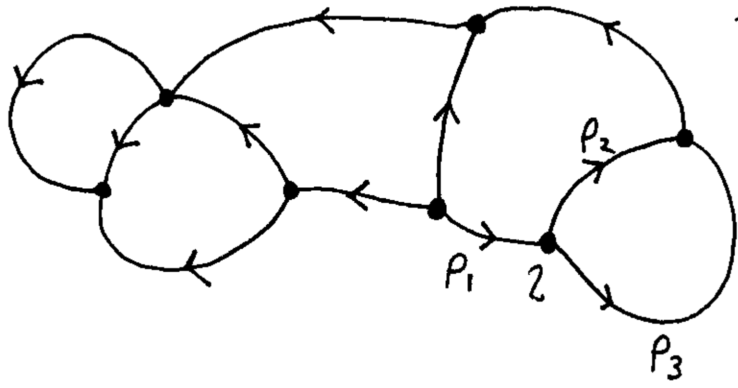
TOWARDS A SPIN FOAM MODEL OF QUANTUM GRAVITY



John Baez 10/11/05

http://math.ucr.edu/home/baez/loops05

SPIN NETWORKS

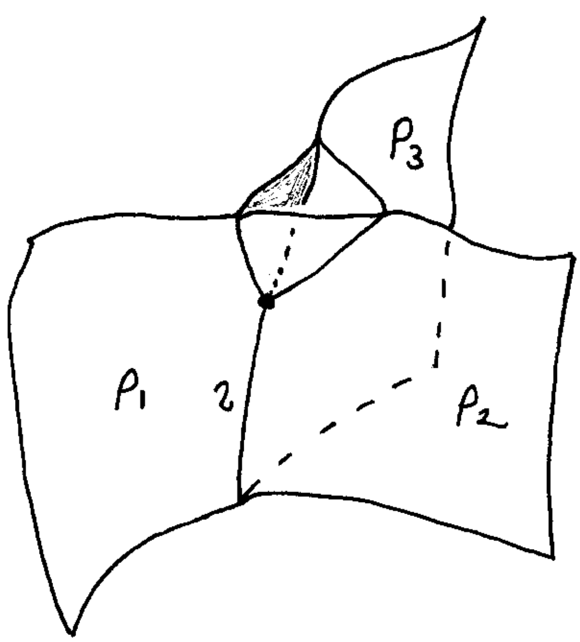


$\iota : \rho_1 \to \rho_2 \otimes \rho_3$

graphs with: edges labelled by group irreps

vertices labelled by intertwining operators

SPIN FOAMS



$\iota : \rho_1 \to \rho_2 \otimes \rho_3$

2-complexes with: faces labelled by group irreps

edges labelled by intertwining ops

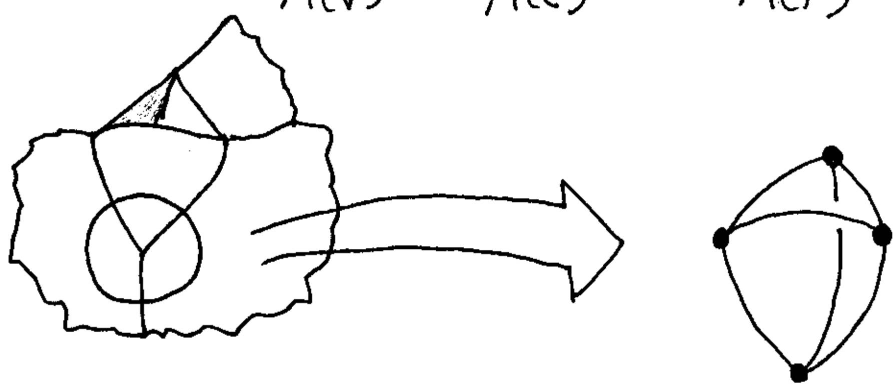
SPIN FOAM MODELS

A spin foam model picks out:

- a group
- allowed irreps, intertwining ops & vertices
- formulas for vertex, edge & face amplitudes

$A(v) \quad A(e) \quad A(f)$

standard choice:



$= A(v)$

Amplitude of a spin foam F:

$$A(F) = \prod_v A(v) \prod_e A(e) \prod_f A(f)$$

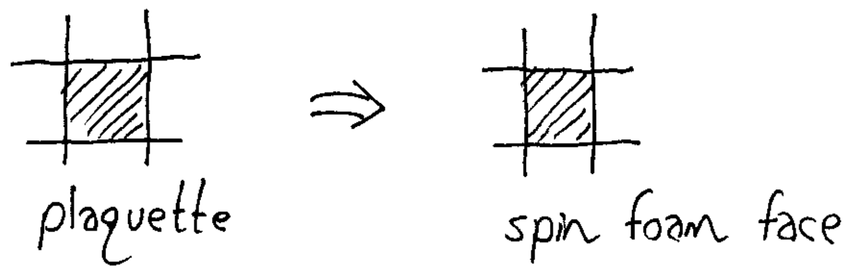
An observable $O(F)$ has expectation value:

$$\langle O \rangle = \frac{\sum_F O(F)\, A(F)}{\sum_F A(F)}$$

← partition function

EXAMPLES OF SPIN FOAM MODELS

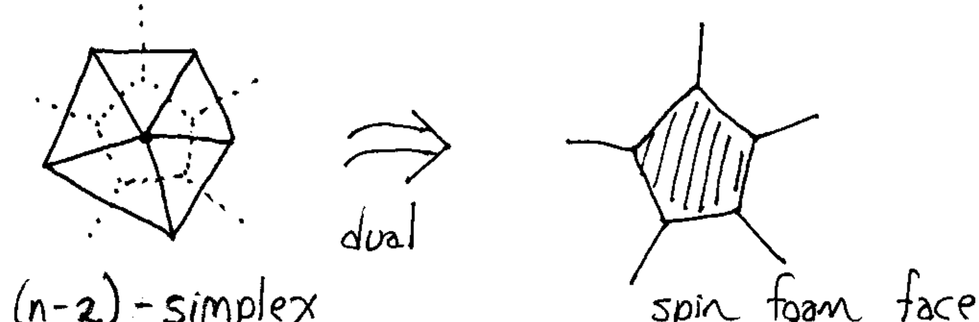
- Lattice gauge theories
  - fixed spin foam topology



$$\int \prod_e dg_e \Rightarrow \sum_F A(F)$$

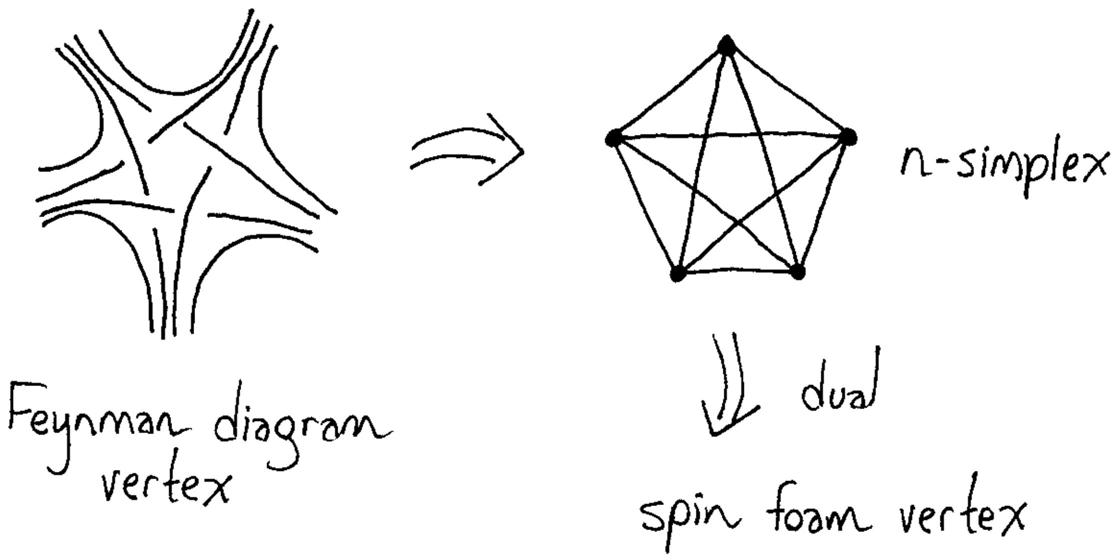
e.g. Yang-Mills, BF theory, Barrett-Crane model
Conrady gr-qc/0504059

- (Old) dynamical triangulations theories
  - trivial gauge group



• "Chain mail" quantum field theories

— spin foams from Feynman diagrams



e.g. Ponzano-Regge, Barrett-Crane models

For reviews see:

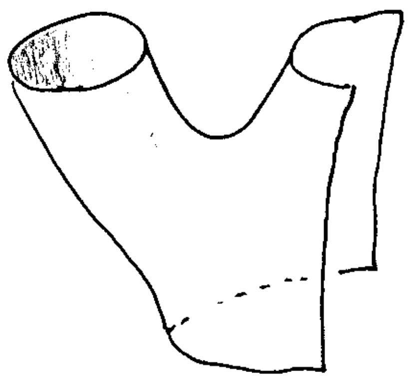
Oriti gr-qc/0311066

Perez gr-qc/0301113

Rovelli <u>Quantum</u> <u>Gravity</u>, Cambridge U.P.

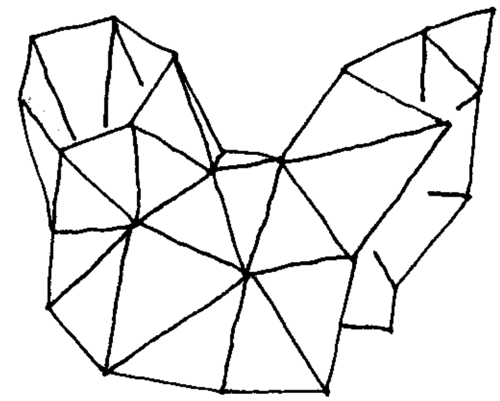
(also on his webpage)

- Topological string theories
  - only planar spin foam vertices
  - amplitudes are purely topological



string worldsheet

$\Longrightarrow$ triangulate



$\Downarrow$ dual

spin foam

Lauda & Pfeiffer –

see Pfeiffer's talk!

GOALS :

1) Interpret spin foams as "quantum geometries" ✓

2) Find a spin foam model where the sum over quantum geometries peaks near solutions of Einstein's equations in the large-scale limit.

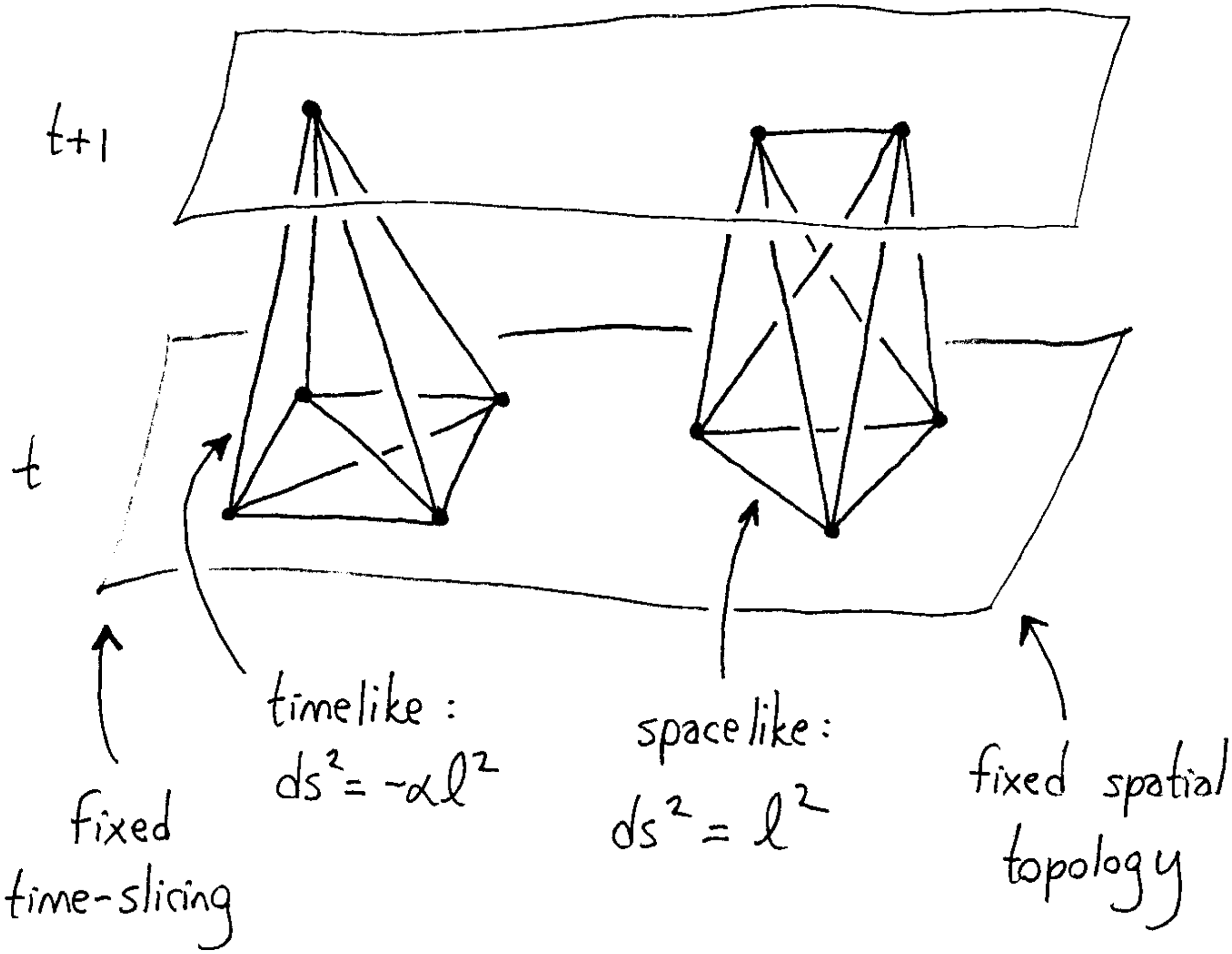
$64,000

QUESTIONS :

- Do we need special <u>matter</u> for 2) to work?
- Do we <u>need</u> to take a continuum limit? ... or need to <u>not</u> take it?

CAUSAL DYNAMICAL TRIANGULATIONS

suggests the answers are all <u>NO</u>!



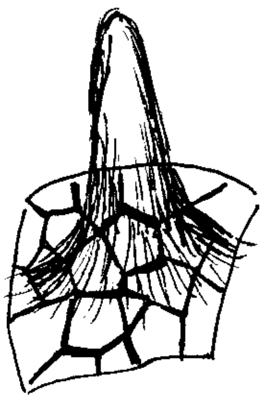
Use Regge action ; Wick rotation

Ambjørn, Loll & Jurkiewicz
hep-th/0505154

Tuning parameters suitably,
take limit as $\ell \to 0$ ...

GOOD RESULTS!

... but surprisingly:



LARGE-SCALE SPECTRAL DIMENSION OF SPACETIME =

$4.02 \pm .1$

SMALL-SCALE SPECTRAL DIMENSION =

$1.80 \pm .25$

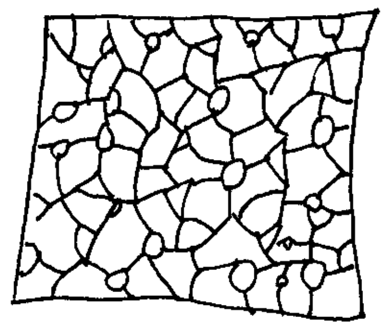
spectral dimension: measured by heat flow

Lauscher, Reuter & others ALSO see signs of small-scale 2d behavior in perturbative QG!

hep-th/0108040

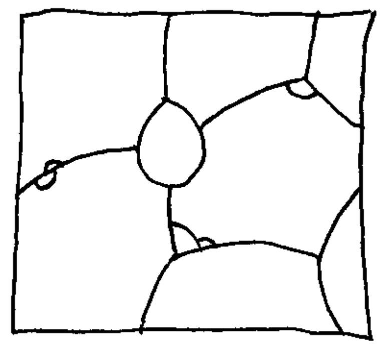
CAN WE GET AT THIS 2D BEHAVIOR "FROM THE BOTTOM UP" VIA SPIN FOAMS?

LARGE SCALES:
$L \gg \ell_p$



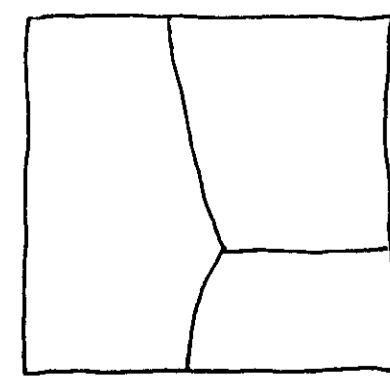
$\dim \cong 4$

INTERMEDIATE SCALES:
$L \cong \ell_p$



$2 < \dim < 4$

SMALL SCALES:
$L \ll \ell_p$



$\dim \cong 2$

(No need for shortest length scale, but possible.)

QUESTION :

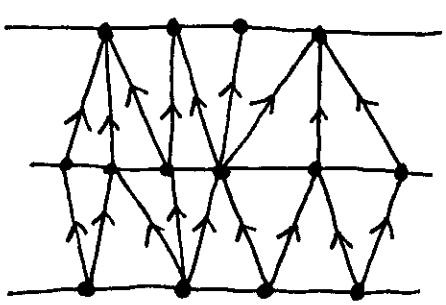
Does causal dynamical triangulations reduce to general relativity as $\ell, \hbar \to 0$? Or does the fixed time-slicing mess things up?

Can we find models <u>without</u> a fixed time-slicing that work like causal dynamical triangulations?

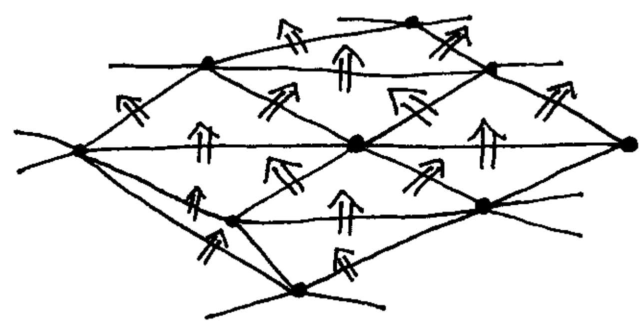
Ashtekar - Marolf - Mourao - Thiemann :

Wick rotation doesn't require a <u>fixed</u> time-slicing.

(quant-ph/9904094)



vs.

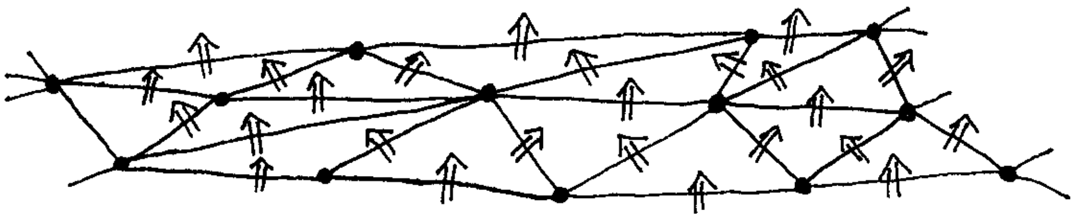


one slicing ; space like vs. timelike edges

many slicings ; partial order on simplices

To get many time-slicings & no topology change, we can use a <u>causal</u> spin foam model —

Markopoulou & Smolin gr-qc/9712067



- n-simplices form a causal set
  =
  spin foam vertices, i.e. <u>events</u>

- nearest future/past neighbors of an n-simplex describe Pachner moves:

 1-2

 2-1 not 

or  !

— so, no topology change!

In 4d one explicit model of this sort is the <u>causal</u> <u>Barrett</u>-<u>Crane</u> model:

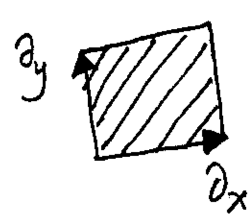
Livine & Oriti gr-qc/0210064

Like the ordinary Barrett-Crane model, this starts with the Lorentz group — what else?

infinitesimal rotations/boosts $\rightarrow$ $\mathfrak{so}(3,1) \cong \Lambda^2 \mathbb{R}^4$ $\leftarrow$ bivectors (or 2-forms)

$$E^i_j \longmapsto E^{ij}$$

$\longmapsto$



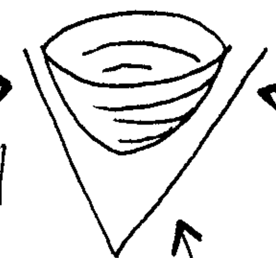
infinitesimal rotation in xy plane

area element $\partial_x \wedge \partial_y$

So, we can <u>quantize</u> bivectors using Lorentz commutation relations! $\quad E^{ij} \rightsquigarrow \hat{E}^{ij}$

HILBERT SPACE OF A "QUANTUM AREA ELEMENT"

We can represent $\hat{E}^{ij}$ as <u>operators</u> on $L^2(H)$:



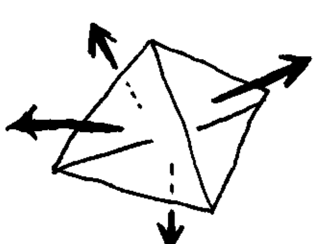
$$H = SO(3,1)/SO(3)$$

connected component

HILBERT SPACE OF A "QUANTUM TETRAHEDRON"

$$\{\Psi \in L^2(H)\otimes L^2(H)\otimes L^2(H)\otimes L^2(H) :$$
$$(\hat{E}_1^{ij} + \hat{E}_2^{ij} + \hat{E}_3^{ij} + \hat{E}_4^{ij})\Psi = 0\}$$

$$\cong L^2(SO(3,1)\backslash SO(3,1)^4/SO(3)^4)$$



$$E_1^{ij} + E_2^{ij} + E_3^{ij} + E_4^{ij} = 0$$

This has (continuous) basis given by <u>area 4-tuples</u>:

SECOND QUANTIZATION

Now treat

$$\psi \in L^2(SO(3,1)\backslash SO(3,1)^4 / SO(3)^4)$$

as a "field"

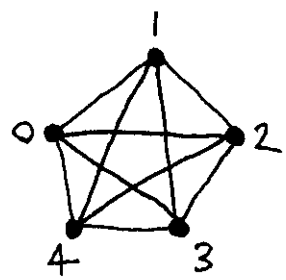
$$\psi: SO(3,1)^4 \to \mathbb{C}$$

and quantize with action:

$$S(\psi) = \frac{1}{2}\int_{SO(3,1)^4} \psi(g_1, g_2, g_3, g_4)^2 +$$

$$\frac{\lambda}{5!}\int_{SO(3,1)^{10}} \psi(g_{01}, g_{02}, g_{03}, g_{04}) \cdot$$

$$\psi(g_{01}, g_{12}, g_{13}, g_{14}) \cdot$$



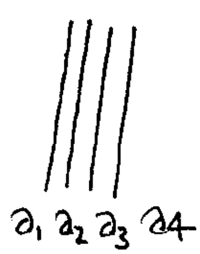
$$\psi(g_{02}, g_{12}, g_{23}, g_{24}) \cdot$$

$$\psi(g_{03}, g_{13}, g_{23}, g_{34}) \cdot$$

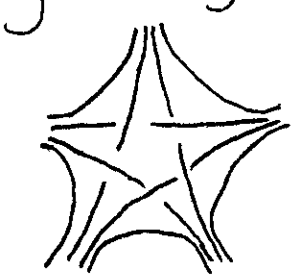
$$\psi(g_{04}, g_{14}, g_{24}, g_{34})$$

Obtain: $\int e^{iS(\psi)} \mathcal{D}\psi = \sum$ Feynman diagrams

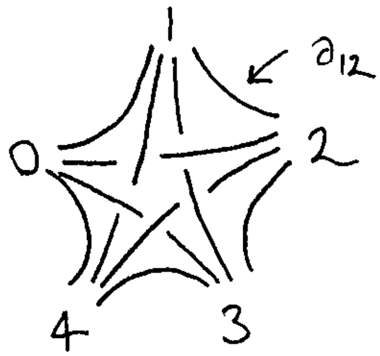
propagator:



interaction:



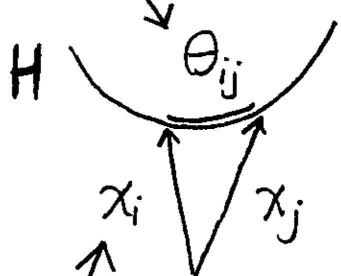
RESULT: the vertex amplitude is



a_{12} – triangle area

$$= \int_{H^5} dx_0 \cdots dx_4 \prod_{0 \le i < j \le 4} \frac{\sin a_{ij}\theta_{ij}}{a_{ij}\sinh\theta_{ij}}$$

dihedral angle



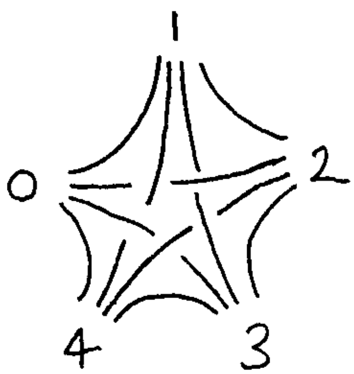
tetrahedron normal

BUT, using causal structure we can say whether ith tetrahedron lies in <u>future</u> or <u>past</u> of jth one & replace $\sin a_{ij}\theta_{ij}$ by <u>one</u> <u>term</u>:

$$e^{\pm i a_{ij}\theta_{ij}}$$

This gives the <u>Regge</u> <u>action</u>!!

CAUSAL VERTEX AMPLITUDE :



=

REGGE ACTION!

$$\int_{H^5} dx_0 \cdots dx_5 \left(\prod_{0 \le i < j \le 4} \pm \frac{1}{a_{ij} \sinh \theta_{ij}} \right) e^{i \sum \pm a_{ij} \theta_{ij}}$$

PROBLEM : PEAKED AT "DEGENERATE 4-SIMPLICES" !

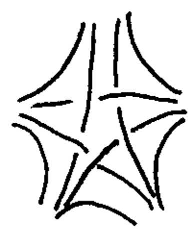
POSSIBLE SOLUTION : DEGENERATE SIMPLICES NEGLIGABLE IN REAL PHYSICS, E.G. GRAVITON SCATTERING?

Rovelli gr-qc/0508124

BUT:

First let's get our hands on a spin foam model that reduces to GR at large length scales ... & THEN let's make it pretty!

For example: why not

 $= e^{i\,(\text{Regge action})}$?