The Problem of Dynamics in Quantum Gravity

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Workshop on Quantum Gravity in the Americas

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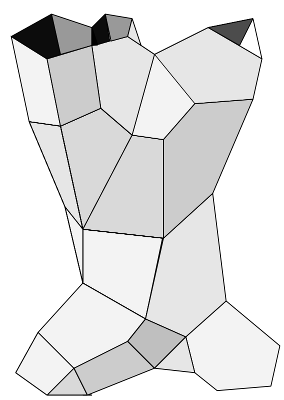


The problem of *dynamics* in quantum gravity is still a big challenge. We don't know how to make spacetime a truly dynamical entity with local degrees of freedom while taking quantum theory into account:

- String theory has discovered a vast 'landscape' of vacua. Each describes a background geometry in which strings can propagate. Perturbative quantization about these vacua seems to be well-behaved — thanks in large part to supersymmetry. Some vacua are related by dualities, and one can study continuous *adiabatic* motions through the space of vacua (e.g. 'flop and conifold transitions'). But nobody knows how any of these vacua can describe our universe unless the big bang is 'just a phase' that our universe will outgrow when it tunnels from its current metastable state to a true vacuum. Supersymmetry breaking is also poorly understood.

- Loop quantum gravity has found a background-free kinematics with interesting properties, but no choice of dynamics has been shown to reduce to general relativity in a suitable limit. This is true both in the old canonical approach and the new 'spin foam' approach. In the canonical approach, extra symmetry assumptions can reduce the problem to one with finitely many degrees of freedom: 'loop quantum cosmology'. Then a *variety* of choices of dynamics give reasonable results.

- Causal dynamical triangulations uses a preferred time-slicing and gives a formula for the 'one-time-step' transition amplitude between two triangulations of space. This dynamics seems to have a well-behaved spacetime as its ground state (vacuum). However, the preferred time-slicing means that extra work is required to show the theory reduces to Einstein's equations in the large-scale limit — rather than some theory in which t plays a distinguished role.

As far as I can tell, nobody has yet found:

A background-free quantum theory with
local degrees of freedom propagating causally.

So, to make progress we don't need a 'theory of everything' — nor even a theory that reduces to general relativity in the large-scale limit! We just need theories that are:

- **Background-free:** not relying on fixed geometrical structures, e.g. fields that appear in the Lagrangian but are not variable, such as a fixed 'background metric'. (So: not perturbative quantum gravity, string theory in any background, dynamical triangulations with a preferred time-slicing.)
- **Quantum:** subject to the uncertainty principle. (So: not classical general relativity.)

and have

- **Local degrees of freedom propagating causally:** some nontrivial sense of 'region' and 'causal shadow' such that observables living in a region R can be computed from observables living in the region S if R is in the causal shadow of S. (So: not TQFTs, 3d quantum gravity).

Warning: these could be just *emergent properties.*

Loop Quantum Gravity

To reach dynamics in loop quantum gravity involves a long road full of choices. Ideally, we should:

1. Choose a description of kinematical states and observables.
2. Choose formulas for diffeomorphism and Hamiltonian constraints.
3. Solve the constraints to find 'physical states'.
4. Find 'physical observables'.
5. Find 'semiclassical states' — physical states for which physical observables have values close to the values of known classical observables in known *classical* states.
6. Calculate physical observables on semiclassical states.

We'd like feedback saying we're on the right track before the very end! The LOST theorem suggests that *if* we take SU(2) holonomies as kinematical observables, there's a unique good kinematical Hilbert space. Area and volume operators on this space are *fairly* well behaved.

But, this gives a setup in which all formulas for the Hamiltonian constraint seem contrived. Reason: this constraint involves not just holonomies but *curvature!* Not enough results showing these formulas have nice properties... and no luck so far getting constraints that satisfy the Dirac algebra.

This makes me worry:

- Is the Ashtekar–Barbero SU(2) connection the best one to use? Why not a Lorentz or chiral spin connection? Are there no-go theorems ruling these out? (Willis)
- Shouldn't curvature be an observable too? Why not try a setup with a less impoverished vocabulary? Observables for surfaces as well as loops?

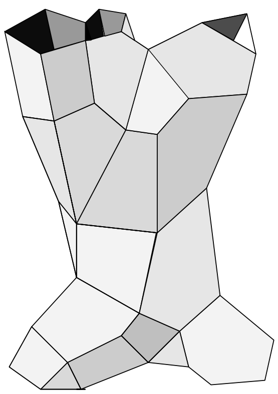
We shouldn't be scared to tinker with the formalism... nor scared to put more work into the existing formalism!

Short of radical rethinking, here are some ways to get feedback soon:

- Quantize the theory after adding extra symmetry assumptions: quantum cosmology. Perturb about these symmetric solutions?
- Study coherent states peaked at classical solutions, but in which the constraints hold only *approximately*.
- Relate loop quantum gravity to perturbative quantum gravity. Gravitons should emerge in some limit; let's get them soon! (Carrión Álvarez)

Spin Foam Models

The Idea: spacetime and everything in it is a quantum superposition of 'spin foams'. A spin foam is a generalized Feynman diagram where instead of a graph we use a higher-dimensional complex:



A spin foam model specifies a class of complexes and labels for vertices, edges, faces, etc. It also says how to calculate an amplitude for any such spin foam — typically as a product of vertex amplitudes, edge amplitudes, face amplitudes, etc.

The Goal: to supplement the Hamiltonian/canonical approach to loop quantum gravity with a Lagrangian/path-integral approach, bringing dynamics into the game sooner. Just as spin networks describe 3d quantum geometry, spin foams should describe 4d quantum geometry!

The road to dynamics via spin foams should go something like this:

1. Choose a description of 3-geometries using spin networks and 4-geometries using spin foams. Superpositions of spin networks are 'kinematical states'; superpositions of spin foams are 'quantum histories'.

2. Choose formulas for amplitudes associated to vertices, edges, faces, etc.

3. Find a sum over spin foams that implements the projection onto physical states **AND/OR** a sum over spin foams of a certain 'timelike thickness' that implements proper-time evolution when applied to kinematical states.

4. Find 'semiclassical 3-geometries': physical states that approximate *classical* metric-connection pairs satisfying the constraints.

5. Calculate time evolution on semiclassical states.

Much less energy has been put into the foundational questions here than for canonical quantum gravity. The problem of time isn't gone, just different. Lower-dimensional toy models can help *if they have local degrees of freedom* — 3d quantum gravity is beautiful, but dangerous.

The Barrett–Crane Model

This model was motivated by the Plebanski action for GR:

$$S = \int \epsilon_{ijk\ell}\, B^{ij} \wedge F_{k\ell} + \phi_{ijk\ell}\, B^{ij} \wedge B^{k\ell}$$

where i, j, k, ℓ are internal indices, F is the curvature of an $\mathrm{SO}(3,1)$ connection, B is an $\mathfrak{so}(3,1)$-valued 2-form, and ϕ is a Lagrange multiplier field such that $\delta S/\delta\phi = 0$ ensures that $B^{ij} \wedge B^{k\ell}$ has the same symmetries as it does when B is built from a cotetrad:

$$B^{ij} = e^i \wedge e^j$$

If B is built from a cotetrad this way, S equals the Einstein-Hilbert action. But, there are other possibilities:

$$B^{ij} = \begin{cases} \pm e^i \wedge e^j \\ \\ \pm \epsilon^{ijk\ell}(e_k \wedge e_\ell) \end{cases}$$

when B is nondegenerate... and still others when B is degenerate!

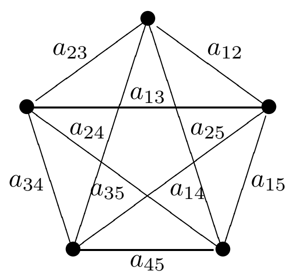
Without the Lagrange multiplier field to constrain the B field, the Plebanski action reduces to that of 'BF theory':

$$S = \int \epsilon_{ijk\ell} B^{ij} \wedge F_{k\ell}$$

BF theory has no local degrees of freedom, and has a well-understood spin foam quantization. So, to get the Barrett–Crane model we impose constraints on the spin foams being summed over, which are analogous to the constraints on B in the Plebanski theory.

To cut a long story short... we obtain a model where for each way of sticking 4-dimensional simplices together along their tetrahedral faces, we get a spin foam after we label each triangle by an area $a \geq 0$.

How do we calculate the amplitude for such a spin foam? First we compute an amplitude for each 4-simplex by doing an integral. A 4-simplex has 10 triangles labelled by areas, and 'morally' this integral is:



$$= \int_{H^5} \prod_{k<\ell} K_{a_{k\ell}}(\phi_{k\ell}) \, \frac{dh_1}{2\pi^2} \cdots \frac{dh_5}{2\pi^2}$$

where the hyperboloid

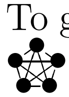
$$H = \{t^2 - x^2 - y^2 - z^2 = 1, \; t > 0\}$$

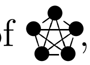
is equipped with its usual Lorentz-invariant measure, ϕ_{kl} is the hyperbolic distance between points $h_k, h_\ell \in H$, and

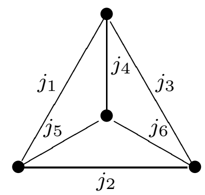
$$K_a(\phi) = \frac{\sin a\phi}{\sinh \phi}$$

is the integral kernel for projecting $L^2(H)$ down to the subspace of functions with $\nabla^2 f = -(a^2 + 1) f$. The points $h_1, \ldots, h_5$ describe the *timelike normal vectors* of the 5 faces of a 4-simplex with all spacelike triangles.

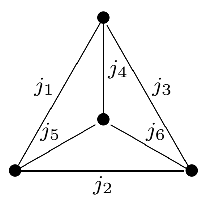
However, this integral diverges because of Lorentz symmetry! So, we 'gauge-fix' it, holding one point h_k fixed.

To get an amplitude for our spin foam, we take a product of these  's over all 4-simplices... and perhaps some other fudge factors for triangles and tetrahedra.

All this is very much like the Ponzano–Regge model for Riemannian $3d$ quantum gravity. Instead of , they used the $6j$ symbol:



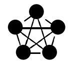
A stationary phase approximation shows that



$$\sim \quad \cos(S + \frac{\pi}{4}) \sqrt{\frac{2}{3\pi V}}$$

in the limit where we rescale all spins to infinity. Here S is the Regge action of the dual tetrahedron with edge lengths $2j_k + 1$, and V is its volume.

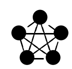
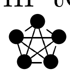
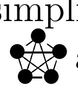
This is *just the start* of relating the Ponzano–Regge model to 3d quantum gravity... but it's promising.

We hoped a similar stationary phase approximation would relate the $10j$ symbols to the Regge action for 4d gravity. But in fact, the asymptotics of  are dominated not by the stationary phase points but by **degenerate 4-simplices**, whose triangles have normal vectors $h_1, \ldots, h_5 \in H^3$ that are almost parallel!

In the Barrett–Crane model 4-simplices are like cotetrads e^i. Describing 4-simplices using their triangle areas is like describing e^i using $B^{ij} = e^i \wedge e^j$. Degenerate 4-simplices are like degenerate cotetrads.

So: perhaps the Barrett–Crane model is correctly quantizing the Plebanski theory, but *the path integral for this theory is dominated by degenerate B fields*, not those coming via $B^{ij} = e^i \wedge e^j$ from a nondegenerate cotetrad.

It's hard to tell if this is bad! Some thoughts:

- If our criterion for success were getting  to have certain asymptotics in terms of the Regge action, we could simply *define* a new  having those asymptotics. Our actual criterion should be getting well-behaved dynamics.
- Realistic dynamics presumably involves spin foams with lots of small 4-simplices, not a few big ones. So, the asymptotic behavior of  as triangle areas $\to \infty$ may not be decisive.

- In the Barrett–Crane model, the 'fudge factors' for triangles and tetrahedra determine whether large or small 4-simplices dominate the sum over spin foams.

  For example, compare the partition function of S^4 triangulated as the boundary of a 5-simplex in two versions of the Riemannian Barrett-Crane model:

J	$Z_J(M)$
0	$1.000 \cdot 10^0$
1/2	$3.722 \cdot 10^5$
1	$7.812 \cdot 10^9$
3/2	$2.128 \cdot 10^{13}$
2	$1.345 \cdot 10^{16}$

DePietri–Freidel–Krasnov–Rovelli model with spin cutoff J

J	$Z_J(M)$
0	1.000000000000
1/2	1.000014319178
1	1.000014323656
3/2	1.000014323670
2	1.000014323670

Perez–Rovelli model with spin cutoff J

- To study spin foams with lots of small 4-simplices, we either need new approximation methods or computer simulations. *We need to get our hands dirty and see what works!* Christensen and Egan's computer simulations are the only way we got this far.

- If we're studying spin foam models with lots of 4-simplices with amplitudes nicely related to the Regge action, we're getting pretty close to the Ambjorn–Jurkiewicz–Loll work on causal dynamical triangulations, so we should borrow ideas from them:

  - Introduce a causal structure — with or without a time slicing — with or without a *preferred* one. If we have a time slicing, preferred or not, we can use Wick rotation.
  - Livine–Oriti: a partial ordering of 4-simplices lets us replace

    $$K_a(\phi) = \frac{e^{ia\phi} - e^{-ia\phi}}{i \sinh \phi}$$

    by a 'propagator' consisting of *one of the two terms*, depending on the causal relation between the two 4-simplices sharing the triangle whose area is a. We should use this not to compute the projection onto physical states, but time evolution: it's like a time-ordered exponential.

- We should also study *spin foam perturbation theory*: perturbative quantum gravity where the excitations are described using spin foams rather than gravitons. See the work of Freidel and Starodubtsev on perturbation about the Kodama state: they get manifestly diffeomorphism-invariant power series in the cosmological constant ($\Lambda \approx 10^{-122}$) where the terms are spin foams involving the q-deformed deSitter group! Mathematically beautiful, but what does it mean?

Again: any ***background-free quantum theory with local degrees of freedom propagating causally*** would be a good thing!