1. Solution to the in-class problem: Suppose our lake of fish with initial population of 100 and growth rate of 0.5 is subject to harvesting: 40 fish are removed each year. Solve the differential equation modeling this situation

\[ \frac{dP}{dt} = kP - h \]

and use it to predict the number of fish after 5 years.

Plugging in \( k = 0.5 \) and \( h = 40 \), we get the differential equation

\[ \frac{dP}{dt} = 0.5P - 40. \]

Separating variables and taking the integral of each side gives

\[ \int \frac{dP}{-0.5P - 40} = \int dt. \]

Integrating gives

\[ 2 \ln |0.5P - 40| = t + c. \]

We divide both sides by 2 to get

\[ \ln |0.5P - 40| = \frac{1}{2}(t + c). \]

Taking \( e \) to both sides, we get

\[ |0.5P - 40| = e^{\frac{1}{2}(t+c)}. \]

Removing the absolute values, we get

\[ 0.5P - 40 = \pm e^{\frac{t}{2}}e^{\frac{c}{2}}. \]

Letting

\[ A = \pm e^{\frac{c}{2}}, \]

we have

\[ 0.5P = Ae^{\frac{t}{2}} + 40. \]
Multiplying both sides by 2, we get

\[ P = 2Ae^{t^2 + 80}. \]

Now, we use our initial population of 100 to find \( A \):

\[ 100 = 2Ae^0 + 80 \]

which gives \( A = 10 \). Thus, our solution is

\[ P(t) = 20e^{t^2} + 80. \]

Plugging in \( t = 5 \), we get

\[ P(5) = 20e^{0.5(5)} + 80 \approx 324 \text{ fish.} \]