Math 9C Homework 19
Commonly Asked Questions

1. Solution to the in-class problem: Show that the function
   \[ y = \frac{\ln(x) + c}{x} \]
   is a solution of the differential equation
   \[ x^2 y' + xy = 1. \]
   Applying the quotient rule, we get that
   \[ y' = \frac{1/x(x) - (\ln(x) + c)}{x^2} = \frac{1 - \ln(x) - c}{x^2}. \]
   Plugging into the differential equation, we get
   \[ x^2 \left( \frac{1 - \ln(x) - c}{x^2} \right) = x \left( \frac{\ln(x) + c}{x} \right) = 1 \]
   which simplifies to
   \[ 1 - \ln(x) - c + \ln(x) + c = 1 \]
   which gives \( 1 = 1 \). Thus, this function is a solution.
Find the solution satisfying \( y(1) = 2 \).
Here, we use this initial condition to find a specific value of \( c \). Plugging in \( x = 1 \) and \( y = 2 \), we get
   \[ 2 = \frac{\ln(1) + c}{1} = c. \]
Thus, \( c = 2 \), and we get the solution
   \[ y = \frac{\ln(x) + 2}{x}. \]

2. What is the purpose of the methods in this section? (question to be answered)
The purpose here is NOT to find a solution. The idea is that, even if we cannot find a solution, we can find good approximations for solutions. Direction fields give us an idea what the graph of a solution looks like. Euler’s method gives an estimate for the the answer we’d get if we plugged in a specific value to a solution.